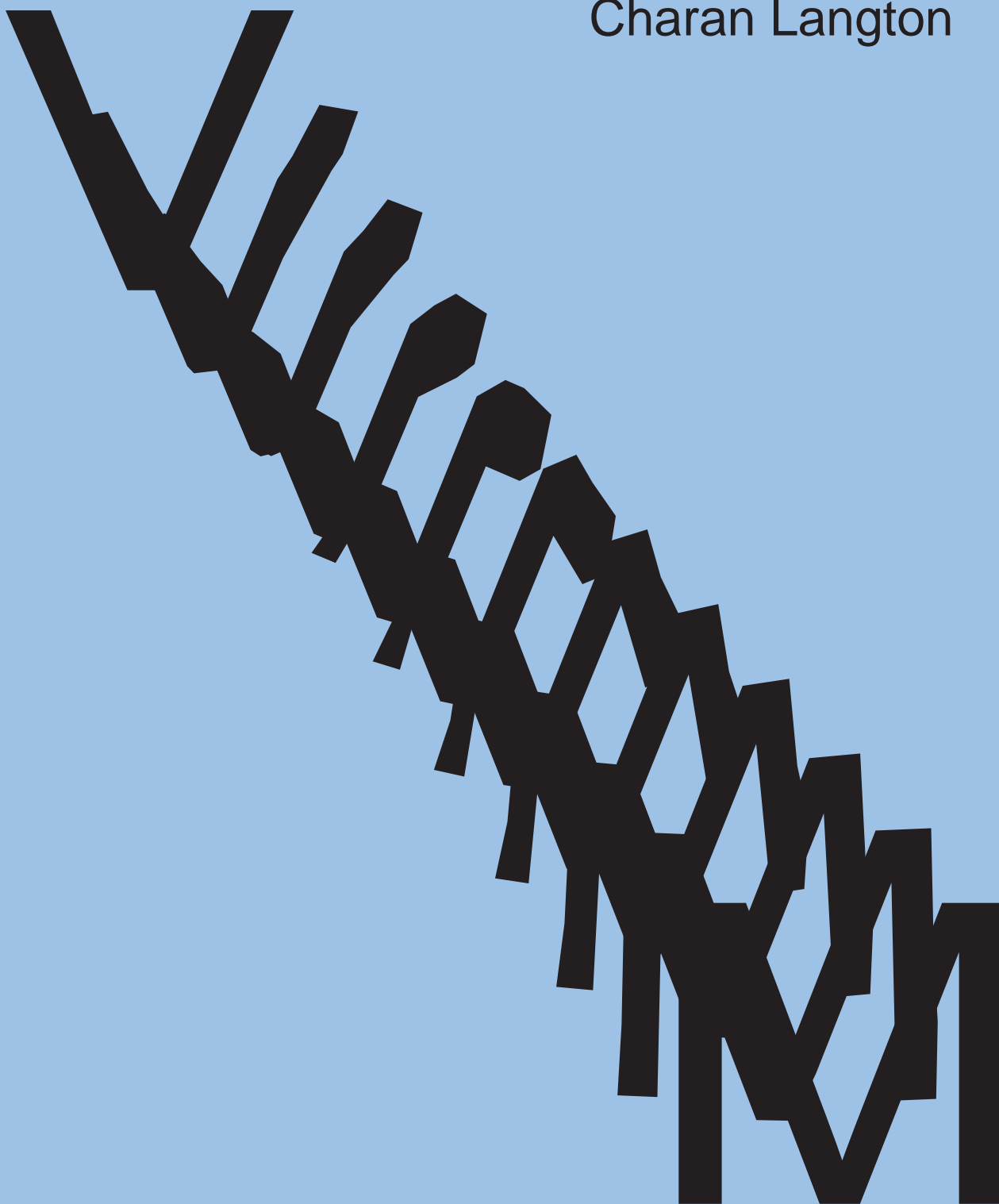


Verbal Math

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1.3 Lesson 3 - *Eighths and Ninths*

IF anything is divided into 8 equal parts, what is one of these parts called?

2. What are 2, 3, 4, 5, 6 and 7 of these parts called, and how many eighths in a unit?

Solution—2 eighths, 3 eighths, 4 eighths, 5 eighths, 6 eighths, 7 eighths;

3. What is one eighth of 24? 48? 72? 88?

4. What are 2 eighths of 32? 40? 56? 72?

5. What are 3 eighths of 16? 64? 80? 32?

6. What are 5 eighths of 8? 24? 48? 64?

7. 2 eighths of 24 is how many times 3?

8. 3 eighths of 40 is how many times 5?

9. 4 eighths of 80 is how many times 8?

10. 5 eighths of 56 is how many times 7?

Solution—One eighth of 56 is 7. 5 eighths is 5 times that, or 35, which is 5 times 7.

11. 6 eighths of 64 is how many times 12?

12. 7 eighths of 72 is how many times 3?

13. 3 eighths of 32 is how many times one third of 12?

One eighth of 32 is 4, and 3 eighths is 3 times 4, or 12.

One third of 12 is 4; The first part is 12, the second part of the problem is 4. 12 is 3 times 4.

14. 6 eighths of 40 is how many times one fourth of 24?

15. 4 eighths of 48 is how many times 2 thirds of 18?

16. 7 eighths of 96 is how many times 3 fifths of 10?

17. 5 eighths of 56 is how many times 5 sixths of 42?

18. 2 thirds of 27 is how many times 3 fourths of 12?

19. If a single thing is divided into 9 equal parts, what are 1, 2, 3, 4, etc., of these parts called?

20. What is 2 ninths of 18? 27? 45? 36?

21. What is 3 ninths of 63? 72? 81? 27?

22. What is 4 ninths of 9? 36? 54? 81?

23. What is 5 ninths of 54? 72? 63? 27?

24. What is 6 ninths of 81? 18? 36? 90?

25. What is 7 ninths of 18? 99? 27? 108?

26. 3 times 6, and 2 thirds of 6, is how many?

27. 4 times 12, plus 3 fourths of 12, is how many?

28. 5 times 10, and 3 fifths of 10, is how many?

29. 6 times 12, and 3 sixths of 12, is how many?

30. 5 times 7, and 4 sevenths of 7, is how many?

31. 9 times 8, and 5 eighths of 8, is how many?

32. 2 times 18, and 7 ninths of 18, is how many?

33. 2 ninths of 18 is how many times 2 thirds of 3?

Solution—2 ninths of 18 is 4; 2 thirds of 3 is 2; 4 is as many times 2 as 2 is contained times in 4, or 2.

34. 5 ninths of 27 is how many times 5 sixths of 6?

35. 6 ninths of 54 is how many times 4 fifths of 15?

36. 3 ninths of 72 is how many times 2 eighths of 16?

37. 7 eighths of 24 is how many times 7 eighths of 8?

38. A bought 15 songs, and sold 6 of them, and then had 4 less than 20; how many did he have at first?

39. Hill and Oliver each had 26 dollars; after Hill had given Oliver 10, and Oliver had given Hill 6; how many does each have now?

40. A grocer had 48 baskets of oranges, sold $\frac{2}{4}$ of them to one store, and $\frac{1}{4}$ to another; how many baskets did he sell to each?

41. A bought 60 radios, and sold one third of them to B, and $\frac{3}{5}$ of the remainder to C; how many radios does he have now?

Answers

If anything is divided into 8 equal parts, what is one of these parts called?

Solution—one eighth.

2. What are 2, 3, 4, 5, 6 and 7 of these parts called, and how many eighths in a unit?

Solution—2 eighths, 3 eighths, 4 eighths, 5 eighths, 6 eighths, 7 eighths;

3. What is one eighth of 24? 48? 72? 88?

3; 6; 9; 11.

4. What are 2 eighths of 32? 40? 56? 72?

4; 8; 10; 14;

5. What are 3 eighths of 16? 64? 80? 32?

6; 24; 30; 36.

6. What are 5 eighths of 8? 24? 48? 64?

5; 15; 30; 40.

7. 2 eighths of 24 is how many times 3?

One eighth of 24 is 3 which is 1 times 3.

8. 3 eighths of 40 is how many times 5?

One eighth of 40 is 5 which is 1 times 5.

9. 4 eighths of 80 is how many times 8?

One eighth of 80 is 10. 4 eighths is 4 times that, or 40, which is 5 times 8.

10. 5 eighths of 56 is how many times 7?

Solution—One eighth of 56 is 7. 5 eighths is 5 times that, or 35, which is 5 times 7.

11. 6 eighths of 64 is how many times 12?

One eighth of 64 is 8. 6 eighths is 6 times that, or 48, which is 4 times 12.

12. 7 eighths of 72 is how many times 3?

21.

13. 3 eighths of 32 is how many times one third of 12?

One eighth of 32 is 4, and 3 eighths is 3 times 4, or 12.

One third of 12 is 4; The first part is 12, the second part of the problem is 4. 12 is 3 times 4.

14. 6 eighths of 40 is how many times one fourth of 24?

One eighth of 40 is 5, and 6 eighths is 6 times 5, or 30; one fourth of 24 is 6; 30 is 5 times 6.

15. 4 eighths of 48 is how many times $\frac{2}{3}$ of 18?

2.

16. 7 eighths of 96 is how many times $\frac{3}{5}$ of 10?

14.

17. 5 eighths of 56 is how many times $\frac{5}{6}$ of 42?

1.

18. $\frac{2}{3}$ of 27 is how many times $\frac{3}{4}$ of 12?

2.

19. If a single thing is divided into 9 equal parts, what are 1, 2, 3, 4, etc., of these parts called?

One ninth, 2 ninths, 3 ninths, 4 ninths, etc.

20. What is $\frac{2}{9}$ of 18? 27? 45? 36?

A sold B one third of 60 radios, or 20 radios, and had $60 - 20$, or 40 remaining; 3 fifths of 40 is 24; so he then has remaining $40 - 24$, or 16 radios.

1.4 Lesson 4 - *Addition, Subtraction*

HARRY gave one third of an apple to his brother, and 2 thirds to his sister; how much did he give away?

Solution—3 thirds or all of it.

2. Matthew gave 2 fifths of a peach to Elias, and 3 fifths to Morris; how much did he give to both?
3. James gave 3 sevenths of a melon to Harry, and 4 sevenths to Harvey; how much did he give away?
4. Danny ate 3 eighths of a quart of chestnuts yesterday, and 4 eighths today; how much did he eat in all?
5. Ella gave one fourth of a melon to Phoebe, 2 fourths to Carrie, and 3 fourths to Kate; how much did she give away?
6. Philip gave 2 sixths of a dollar to Jane, 3 sixths to Sarah, and 5 sixths to Eliza; how much did he give away?
7. Willie lost 7 fifths of a dollar, and had 9 fifths of a dollar remaining; how much did he have at first?

Solution—If Willie lost 7 fifths of a dollar and had 9 fifths remaining, before his loss he had $7 + 9$, or 16 fifths of a dollar.

8. Matthew lost 6 eighths of a dollar from one pocket, and 7 eighths from the other, and had 5 eighths remaining; how much did he have at first?

9. Dora gave 3 ninths of a pound of raisins to Ella, and 7 ninths to Daisy, and then had 3 ninths left; how many did she have at first?

10. Jane had 7 eighths of a pound of candies, and gave Maria 5 eighths of a pound; how many eighths remained?

11. Frank had 6 sevenths of a melon, and gave Abram 4 sevenths of a melon; how much remained?
12. Louisa, had 10 eighths of a dollar, and gave Lizzie 7 eighths of a dollar; how much is left?
13. What is the difference between 5 sevenths, and the sum of 4 sevenths and 6 sevenths?
14. Sallie, had 24 pears, and gave Ben 2 eighths and Amanda 3 eighths of them; how many are left?
15. Rolf, had one third of a cup of chestnuts, bought 4 thirds of a cup more, and then sold 1 cup; what part of a cup is left?

Solution—one third + 4 thirds are 5 thirds; 1 quart contains 3 thirds; 5 thirds - 3 thirds are 2 thirds.

16. Peter, had 5 sixths of a basket of apples, he then sold 3 sixths, and then bought 2 sixths of a basket, how many sixths does he have now?

17. What is the difference between the sum of 3 eighths and 7 eighths, and the sum of 4 eighths and 5 eighths?

18. A bought 20 tires, and sold 2 tenths of them to B, 3 tenths to C and 4 tenths to D; how many tires are left?

19. A woman, had 36 meters of tape, sold 5 ninths it to one person, and 3 ninths to another; how-much does she have now?

Solution—5 ninths + 3 ninths is 8 ninths; hence she had remaining 9 ninths - 8 ninths, or one ninth of 36 meters, or

20. Mariana had 3 fourths of a pint of nuts, Elva had twice as many, and Ezra 3 times as many; how many had they all?

21. A bought 4 ninths of a bag of wheat, and B bought 3 times as much; how much did B buy?

22. At 7 fifths of a dollar each, how much will 5 candies cost?

23. Mary, had one fifth of a melon, gave 2 fifths to Sarah, and twice as much to Sophia. How much is left?

Solution—Sarah received 2 fifths and Sophia twice 2 fifths, or 4 fifths; both receive 2 fifths + 4 fifths,

or 6 fifths of a melon; 1 one fifth - 6 fifths is 5 fifths, or 1 melon.

24. Cornell gave 3 times 3 sixths of an apple to Gray, and had 4 times 3 sixths remaining; how much did he have at first?

25. What will one fifth of a meter of tape cost, at the rate of 20 fourths cents a meter? What will 8 fifths cost at the same rate?

26. Stanton, had 2 thirds of a dollar, found one half of 4 thirds of a dollar; how many thirds of a dollar did he have then?

27. Michael bought 7 tenths of a barrel of monkeys, and then sold 2 thirds of 6 tenths of a barrel; how much is left?

28. Thornton, had 8 sixths of a box of paper, bought 3 fourths of 20 sixths of a box; how much does he have now?

29. Ferris lost 6 ninths of a dollar, and then, found 3 ninths of a dollar, and had 3 fourths of 8 ninths of a dollar remaining; how much did he have at first?

Answers

HARRY gave one third of an apple to his brother, and 2 thirds to his sister; how much did he give away?

Solution—3 thirds or all of it.

2. Matthew gave 2 fifths of a peach to Elias, and 3 fifths to Morris; how much did he give to both?

5 fifths or all of it.

3. James gave 3 sevenths of a melon to Harry, and 4 sevenths to Harvey; how much did he give away?

7 sevenths or all of it.

4. Danny ate 3 eighths of a quart of chestnuts yesterday, and 4 eighths today; how much did he eat in all?

7 eighths.

5. Ella gave one fourth of a melon to Phoebe, 2 fourths to Carrie, and 3 fourths to Kate; how much did she give away?

6 fourths.

6. Philip gave 2 sixths of a dollar to Jane, 3 sixths to Sarah, and 5 sixths to Eliza; how much did he give away?

10 sixths.

7. Willie lost 7 fifths of a dollar, and had 9 fifths of a dollar remaining; how much did he have at first?

Solution—If Willie lost 7 fifths of a dollar and had 9 fifths remaining, before his loss he had $7 + 9$, or 16 fifths of a dollar.

8. Matthew lost 6 eighths of a dollar from one pocket, and 7 eighths from the other, and had 5 eighths remaining; how much did he have at first?

If he lost $6 \text{ eighths} + 7 \text{ eighths}$, or 13 eighths of a dollar, and had 5 eighths remaining, he had at first $13 + 5$, or 18 eighths of a dollar.

9. Dora gave 3 ninths of a pound of raisins to Ella, and 7 ninths to Daisy, and then had 3 ninths left; how many did she have at first?

13 ninths.

10. Jane had 7 eighths of a pound of candies, and gave Maria 5 eighths of a pound; how many eighths remained?

2 eighths.

11. Frank had 6 sevenths of a melon, and gave Abram 4 sevenths of a melon; how much remained?

2 sevenths.

12. Louisa, had 10 eighths of a dollar, and gave Lizzie 7 eighths of a dollar; how much is left?

3 eighths.

13. What is the difference between 5 sevenths, and the sum of 4 sevenths and 6 sevenths?

5 sevenths.

14. Sallie, had 24 pears, and gave Ben 2 eighths and Amanda 3 eighths of them; how many are left?
9 pears.

15. Rolf, had one third of a cup of chestnuts, bought 4 thirds of a cup more, and then sold 1 cup; what part of a cup is left?

Solution—one third + 4 thirds are 5 thirds; 1 quart contains 3 thirds; 5 thirds - 3 thirds are 2 thirds.

16. Peter, had 5 sixths of a basket of apples, he then sold 3 sixths, and then bought 2 sixths of a basket, how many sixths does he have now?

5 sixths - 3 sixths is 2 sixths; 2 sixths + 2 sixths is 4 sixths.

17. What is the difference between the sum of 3 eighths and 7 eighths, and the sum of 4 eighths and 5 eighths?

One eighth.

18. A bought 20 tires, and sold 2 tenths of them to B, 3 tenths to C and 4 tenths to D; how many tires are left?

2 tenths + 3 tenths + 4 tenths are 9 tenths; he had remaining 10 tenths - 9 tenths, or one tenth of 20 tires, or 2 tires.

19. A woman, had 36 meters of tape, sold 5 ninths it to one person, and 3 ninths to another; how-much does she have now?

Solution—5 ninths + 3 ninths is 8 ninths; hence she had remaining 9 ninths - 8 ninths, or one ninth of 36 meters, or

4 meters.

20. Mariana had 3 fourths of a pint of nuts, Elva had twice as many, and Ezra 3 times as many; how many had they all?

18 fourths.

21. A bought 4 ninths of a bag of wheat, and B bought 3 times as much; how much did B buy?

12 ninths.

22. At 7 fifths of a dollar each, how much will 5 candies cost?

35 fifths, or \$7.

23. Mary, had 1 one fifths of a melon, gave 2 fifths to Sarah, and twice as much to Sophia. How much is left?

Solution—Sarah received 2 fifths and Sophia twice 2 fifths, or 4 fifths; both receive 2 fifths + 4 fifths, or 6 fifths of a melon; 1 one fifths - 6 fifths is 5 fifths, or 1 melon.

24. Cornell gave 3 times 3 sixths of an apple to Gray, and had 4 times 3 sixths remaining; how much did he have at first?

2 one sixths.

25. What will one fifth of a meter of tape cost, at the rate of 20 fourths cents a meter? What will 8 fifths cost at the same rate?

4 fourths of a cent, or cent; 3 cents. 8 fifths will cost 24 cents.

26. Stanton, had 2 thirds of a dollar, found one half of 4 thirds of a dollar; how many thirds of a dollar did he have then?

4 thirds.

27. Michael bought 7 tenths of a barrel of monkeys, and then sold 2 thirds of 6 tenths of a barrel; how much is left?

3 tenths of a barrel.

28. Thornton, had 8 sixths of a box of paper, bought 3 fourths of 20 sixths of a box; how much does he have now?

23 sixths of a box.

29. Ferris lost 6 ninths of a dollar, and then, found 3 ninths of a dollar, and had 3 fourths of 8 ninths of

a dollar remaining; how much did he have at first?

1 dollar.

1.5 Lesson 5 - *Fraction analysis*

HOW much will 4 apples cost, if 3 apples cost 9 cents?

Solution—If 3 apples cost 9 cents, 1 apple will cost one third of 9 cents, which is 3 cents; and if 1 apple costs 3 cents, 4 apples will cost 4 times 3 cents, which is 12 cents.

2. What will 5 lemons cost, at the rate of 3 for 12 cents?

Solution—One lemon costs $\frac{12}{3} = 4$ cents. So 5 lemons will cost 20 cents.

3. If 3 pairs of shoes cost 60 dollars, how much will 5 pairs cost?

4. If 4 peaches are worth 80 cents, what are 8 peaches worth?

5. What are 10 oranges worth, if 8 oranges cost 160 cents?

6. If 7 pounds of meat costs 420 cents, what will 9 pounds cost?

7. What is the cost of 11 bags of salt, at the rate of 5 bags for 30 dollars?

8. How far will a man travel in 12 days, at the rate of 36 miles in 4 days?

9. How many tons of hay will a rancher need in 11 weeks, at the rate of 10 tons in 5 weeks?

10. What is the cost of 5 ducks, at the rate of \$12 for 3 ducks?

11. Mary paid \$3 for apples, at the rate of \$1 for 9; how many did she buy?

12. Rena paid 8 dollars for lace, at the rate of 5 dollars for 15 meters; how many meters did she buy?

13. Wilkinson walked 7 hours, at the rate of 12 miles in 4 hours; how far did he walk?

14. Robert traded 12 oranges for apples, at the rate of 3 oranges for 9 apples; how many apples did he get?

15. At the rate of 3 melons for 12 oranges, how many oranges can be exchanged for 10 melons?

Solution—You get 4 oranges for a melon. So 10 melons is equal to 40 oranges.

16. If 6 men can mow 12 acres of grass in a day, how much can 8 men mow in the same time?

17. If 10 men can lay 30 meters of pipeline in one day, how much can 12 men lay in the same time?

18. How long will it take 4 men to use a box of tea if 6 men can use it in 12 days?

Solution—If 6 men can drink a box of tea in 12 days, it will take 1 man 6 times 12 days, or 72 days, and 4 men will drink it in one fourth of 72 days, or 18 days.

19. If 5 boys can do a piece of work in 16 days, how long will it take 20 boys to do it?

20. In what time will 8 girls pick a basket of berries if 4 girls can do it in 8 hours?

21. How many machinists will it take to build a car in 6 days, if 3 machinists can do it in 12 days?

6 machinists.

22. How many workers can do as much work in 4 days as 8 workers can do in 40 days?

23. If it requires 10 workers 8 days to build a wall, how many workers will be required to build it in 5 days?

24. If 5 workers build a boat in 20 days, how many workers will be needed to do it in one fourth of the time?

Solution—If 5 workers build a boat in 20 days, to do it in one fourth of the time will require 4 times 5 workers, or 20 workers.

25. If 7 meters of wire cost 21 dollars, what will 2 thirds of 15 meters cost?

26. Mary exchanged 7 apples for 21 chestnuts; at this rate how many chestnuts could she get for 8 apples?

27. If 8 lemons are worth 16 oranges, how many oranges can you get for 10 lemons?

28. At the rate of 6 grapefruits for 18 melons, how many melons may be purchased for 11 grapefruits?

29. If 9 apples are worth 27 chestnuts, how many chestnuts can you get for 12 apples?
30. I can get 4 boxes of popcorn for 6 quarts of apple juice; what do two quarts of apple juice cost, if 4 boxes of popcorn cost \$4.80?
31. Albert can get 5 boxes of popcorn for same price as 2 candy bars; what is a box of popcorn worth, if 3 candy bars cost 3 dollars?
32. Bimla bought 7 teashirts for 21 dollars, and in payment agreed to work at the store for 7 dollars per hour; how many hours did she work?

Answers

HOW much will 4 apples cost, if 3 apples cost 9 cents?

Solution—If 3 apples cost 9 cents, 1 apple will cost one third of 9 cents, which is 3 cents; and if 1 apple costs 3 cents, 4 apples will cost 4 times 3 cents, which is 12 cents.

2. What will 5 lemons cost, at the rate of 3 for 12 cents?
Solution—One lemon costs $\frac{12}{3} = 4$ cents. So 5 lemons will cost 20 cents.
3. If 3 pairs of shoes cost 60 dollars, how much will 5 pairs cost?
One pair of shoes costs $\frac{60}{3} = 20$ dollars. So 5 pairs will cost \$100.
4. If 4 peaches are worth 80 cents, what are 8 peaches worth?
160 cents.
5. What are 10 oranges worth, if 8 oranges cost 160 cents?
200 cents.
6. If 7 pounds of meat costs 420 cents, what will 9 pounds cost?
540 cents.
7. What is the cost of 11 bags of salt, at the rate of 5 bags for 30 dollars?
\$66.
8. How far will a man travel in 12 days, at the rate of 36 miles in 4 days?
108 miles.
9. How many tons of hay will a rancher need in 11 weeks, at the rate of 10 tons in 5 weeks?
22 tons.
10. What is the cost of 5 ducks, at the rate of \$12 for 3 ducks?
\$20.
11. Mary paid \$3 for apples, at the rate of \$1 for 9; how many did she buy?
27 apples.
12. Rena paid 8 dollars for lace, at the rate of 5 dollars for 15 meters; how many meters did she buy?
24 meters.
13. Wilkinson walked 7 hours, at the rate of 12 miles in 4 hours; how far did he walk?
21 miles.
14. Robert traded 12 oranges for apples, at the rate of 3 oranges for 9 apples; how many apples did he get?
36 apples.
15. At the rate of 3 melons for 12 oranges, how many oranges can be exchanged for 10 melons?
Solution—You get 4 oranges for a melon. So 10 melons is equal to 40 oranges.
16. If 6 men can mow 12 acres of grass in a day, how much can 8 men mow in the same time?
16 acres.
17. If 10 men can lay 30 meters of pipeline in one day, how much can 12 men lay in the same time?
36 meters.
18. How long will it take 4 men to use a box of tea if 6 men can use it in 12 days?
Solution—If 6 men can drink a box of tea in 12 days, it will take 1 man 6 times 12 days, or 72 days, and 4 men will drink it in one fourth of 72 days, or 18 days.

$$\frac{27}{18}.$$

12. What is the sum of $\frac{2}{3}$ and $\frac{3}{4}$?

Solution— $\frac{2}{3}$ equals $\frac{8}{12}$, and $\frac{3}{4}$ equals $\frac{9}{12}$; plus $\frac{9}{12}$ are $\frac{17}{12}$. Which equals $1\frac{5}{12}$.

What is the sum?

13. of $\frac{1}{2}$ and $\frac{1}{3}$?

$$\frac{1}{2} = \frac{3}{6} \text{ and } \frac{1}{3} = \frac{2}{6}; \frac{3}{6} + \frac{2}{6} \text{ are } \frac{5}{6}.$$

14. of $\frac{1}{3}$ and $\frac{1}{4}$?

$$\frac{1}{3} = \frac{4}{12} \text{ and } \frac{1}{4} = \frac{3}{12}; \frac{4}{12} + \frac{3}{12} = \frac{7}{12}.$$

15. of $\frac{1}{4}$ and $\frac{1}{5}$?

$$\frac{9}{20}.$$

16. of $\frac{2}{3}$ and $\frac{3}{4}$?

$$1\frac{5}{12}.$$

17. of $\frac{2}{3}$ and $\frac{2}{5}$?

$$1\frac{1}{15}.$$

18. of $\frac{2}{5}$ and $\frac{3}{4}$?

$$1\frac{3}{20}.$$

19. of $\frac{3}{4}$ and $\frac{3}{6}$?

$$1\frac{1}{4}.$$

20. of $\frac{2}{4}$ and $\frac{2}{7}$?

$$\frac{11}{14}.$$

21. of $\frac{5}{6}$ and $\frac{4}{5}$?

$$1\frac{19}{20}.$$

22. of $\frac{1}{4}$ and $\frac{1}{9}$?

$$\frac{13}{36}.$$

23. of $\frac{2}{5}$ and $\frac{3}{7}$?

$$\frac{29}{35}.$$

24. of $\frac{6}{7}$ and $\frac{3}{5}$?

$$\frac{17}{20}.$$

25. of $\frac{5}{6}$ and $\frac{4}{7}$?

$$1\frac{17}{42}.$$

26. of $\frac{3}{7}$ and $\frac{3}{8}$?

$$\frac{45}{56}.$$

27. of $2\frac{1}{2}$ and $3\frac{1}{3}$?

$$5\frac{5}{6}.$$

28. of $3\frac{1}{3}$ and $4\frac{1}{4}$?

$$7\frac{7}{12}.$$

29. of $2\frac{2}{3}$ and $1\frac{3}{4}$?

$$4\frac{5}{12}.$$

30. of $3\frac{1}{4}$ and $2\frac{3}{5}$?

$$5\frac{17}{20}.$$

31. of $6\frac{2}{5}$ and $5\frac{3}{6}$?

$$11\frac{9}{10}.$$

32. of $4\frac{1}{6}$ and $5\frac{1}{7}$?

$$9\frac{13}{42}.$$

33. of $6\frac{1}{5}$ and $5\frac{1}{8}$?

$$11\frac{13}{40}.$$

34. of $7\frac{2}{3}$ and $8\frac{3}{7}$?

$$6\frac{2}{21}$$

35. of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{4}$?

$$1\frac{7}{12}$$

36. of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$?

$$\frac{47}{60}$$

37. of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$?

$$\frac{7}{8}$$

38. James received $\frac{1}{4}$ of a dollar on Monday, and $\frac{1}{5}$ of a dollar on Tuesday; how much did he get in total?

Solution—In the 2 days he will earn $\frac{1}{4}$ of a dollar + $\frac{1}{5}$ of a dollar; $\frac{1}{4} = \frac{5}{20}$ and $\frac{1}{5} = \frac{4}{20}$; $\frac{5}{20} + \frac{4}{20} = \frac{9}{20}$ of a dollar.

39. Jane bought one stamp for $\frac{1}{4}$ of a dollar, and another stamp for $\frac{5}{8}$ of a dollar; what was the cost of both?

The slate and arithmetic together cost $\frac{1}{4}$ of a dollar + $\frac{1}{5}$ of a dollar; $\frac{1}{4} = \frac{4}{20}$; and $\frac{1}{5} = \frac{5}{20}$ dollar; $\frac{5}{20} + \frac{4}{20} = \frac{9}{20}$ of a dollar.

40. A man, had $\frac{2}{5}$ of a barrel of paint, bought $\frac{3}{4}$ of a barrel; how much did he have then?

If he bought $\frac{3}{4}$ of a barrel, he had $\frac{2}{5}$ of a barrel + $\frac{3}{4}$ of a barrel; $\frac{2}{5} = \frac{8}{20}$; and $\frac{3}{4} = \frac{15}{20}$; $\frac{8}{20} + \frac{15}{20} = \frac{23}{20}$, or $1\frac{3}{20}$ barrels.

41. Rachel had $\frac{1}{2}$ of a dozen of pins, found $\frac{5}{6}$ of a dozen; how many pins did she have then?

$$\frac{1}{2} = \frac{3}{6}; \frac{3}{6} \text{ of a dozen} + \frac{5}{6} \text{ of a dozen} = \frac{8}{6}, \text{ or } 1\frac{1}{3} \text{ dozens, or 16 pins.}$$

42. Jane, had a certain sum of money, received another $\frac{3}{4}$ as much; what part of the sum does she have now?

She had $\frac{4}{4}$ of the sum at first; and $\frac{4}{4} + \frac{3}{4} = \frac{7}{4}$ of the sum.

43. If $\frac{1}{2}$ of a number, increased by $\frac{1}{3}$ of the number equals 50, what is the number?

44. Sarah, had 40 daisies, gave $\frac{1}{2}$ of them to her sister and $\frac{1}{3}$ of them to her mother; how many are left?

$$\frac{1}{2} = \frac{3}{6} \text{ and } \frac{1}{3} = \frac{2}{6}; \frac{3}{6} + \frac{2}{6} = \frac{5}{6}; 6 \cdot 6 - \frac{5}{6} = \frac{1}{6}; \text{ of 48 is 8.}$$

45. James' money, increased by its $\frac{4}{5}$, equals 90 cents; how much money did James have?

46. $\frac{5}{5}$ of James's money + $\frac{4}{5}$ of his money = $\frac{9}{5}$ of his money, which is 90 cents; hence his money is 50 cents.

$$\frac{2}{3} = \frac{8}{12} \text{ and } \frac{3}{4} = \frac{9}{12}; \frac{8}{12} + \frac{9}{12} = \frac{17}{12}; \text{ if } \frac{17}{12} \text{ of the number equals 34, the number is 24.}$$

46. $\frac{2}{3}$ of a certain number, increased by $\frac{3}{4}$ of the same number, equals 84; what is the number?

47. Danny's number of stickers, increased by $\frac{1}{2}$ and $\frac{1}{3}$ of his number, equals 55; how many stickers did he have?

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}; \frac{6}{6} + \frac{5}{6} = 1\frac{1}{6}; \text{ If } \frac{11}{6} \text{ of her number equals 55, the number is 30.}$$

48. $\frac{2}{5}$ of Ahmad's money, increased by $\frac{3}{4}$ of his money, equals 69 dollars; how much money did Ahmad have?

$$\frac{2}{5} + \frac{3}{4} = \frac{8}{20} + \frac{15}{20} = \frac{23}{20}; \text{ if } \frac{23}{20} \text{ of his money equals 69, then his money is 60 dollars.}$$

49. Peter, had $\frac{3}{8}$ of a certain sum of money, was paid $\frac{1}{2}$ of the same sum, and then had \$21; how much did he have at the beginning?

$$\frac{3}{8} + \frac{1}{2} = \frac{3}{8} + \frac{4}{8} = \frac{7}{8}; \text{ if } \frac{7}{8} \text{ of the sum is } \$21, \text{ the sum is } \$24.$$

50. Ann gave 24 dollars for a watch, and $\frac{1}{2} + \frac{2}{3}$ of this is 4 times what she paid for the chain; what is the cost of the chain?

$\frac{1}{2} + \frac{2}{3} = \frac{7}{6}$; $\frac{7}{6}$ of 24 dollars, or 28 dollars, equals 4 times the cost of the chain, therefore the chain cost $\frac{1}{4}$ of 28 dollars, or 7 dollars.

2.7 Lesson 7 - Subtraction of Fractions

WHAT is the difference between $\frac{6}{7}$ and $\frac{3}{7}$?

21. $\frac{3}{5}$ of $2\frac{1}{2}$?
 $\frac{3}{2}$.
22. $\frac{5}{6}$ of $2\frac{2}{5}$?
 2.
23. $\frac{3}{5}$ of 6
 $2\frac{3}{5}$.
24. $\frac{2}{5}$ of $3\frac{3}{4}$?
 $\frac{3}{2}$.
25. $\frac{3}{2}$ of 11?
 $16\frac{1}{2}$.
26. $\frac{2}{3}$ of $\frac{3}{4}$ of $1\frac{1}{7}$?
 $\frac{4}{7}$.
27. $\frac{3}{5}$ of \$40 is 2 times what A gave for a calendar; what was the cost of the calendar?
 \$12.
28. $\frac{5}{7}$ of $\frac{14}{25}$ of a dollar is $\frac{1}{10}$ of the cost of a watch; how much did the watch cost?
 \$4
29. B has 27 marbles, and $\frac{2}{3}$ of B's number equals $\frac{2}{5}$ of C's number; how many marbles has C?
 45 marbles.
30. Henry's hat cost $\frac{4}{5}$ of an 20 dollars, which is $\frac{5}{3}$ of $\frac{3}{5}$ of the cost of his coat; what is the cost of his shirt.
 16 dollars.
31. Out of 48 chestnuts, how many does each of two boys receive, if A receives $\frac{6}{8}$, and B receives $\frac{2}{3}$ as many as A?
 A, 36 chestnuts; B, 24.
32. Hanah bought $\frac{4}{6}$ of a box of clips, which is $\frac{5}{8}$ of what Sarah bought; how many clips did each purchase; provided there are 24 clips in box?
 Hanah, 16 ; Sarah, 18.
33. A has 40 fruit trees, $\frac{4}{10}$ of which are apples, $\frac{1}{2}$ of the remainder are pears, and the rest are peaches; how many trees of each kind has he?
 16 apple, 12 pear, 12 peach.
34. Jenson's age, reduced by its $\frac{1}{4}$ and $\frac{1}{5}$, is 22 years, and his age is $\frac{4}{5}$ of his uncle's age; required the age of eac
 $\frac{1}{4} + \frac{1}{5} = \frac{9}{20}$; $\frac{20}{20} - \frac{9}{20} = 1\frac{1}{20}$; if $\frac{11}{20}$ of his age is 22 years, $\frac{20}{11}$ is 40 years; if $\frac{4}{5}$ of his uncle's age is 40 years, his uncle's age is 50 years.

2.9 Lesson 9 - Compound to Simple Fractions

WHAT is $\frac{1}{3}$ of $\frac{1}{4}$?

Solution— $\frac{1}{3}$ of $\frac{1}{4}$ is one of the three equal parts into which $\frac{1}{4}$ may be divided; if each fourth is divided into three equal parts, 4 fourths or the unit will be divided into 4 times 3, or 12 equal parts; hence, each part is $\frac{1}{12}$ of a unit. Therefore, $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$.

2. What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{8}$? $\frac{1}{2}$ of $\frac{1}{10}$?
3. What is $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{6}$?
4. What is $\frac{1}{4}$ of $\frac{1}{6}$? $\frac{1}{6}$ of $\frac{1}{5}$? $\frac{1}{7}$ of $\frac{1}{6}$?
5. What is $\frac{1}{5}$ of $\frac{1}{3}$? $\frac{1}{7}$ of $\frac{1}{2}$? $\frac{1}{7}$ of $\frac{1}{3}$?
6. What is $\frac{1}{4}$ of $\frac{1}{8}$? $\frac{1}{6}$ of $\frac{1}{8}$? $\frac{1}{7}$ of $\frac{1}{9}$?
7. What is $\frac{1}{6}$ of $\frac{1}{12}$? $\frac{1}{3}$ of $\frac{1}{12}$? $\frac{1}{35}$ of $\frac{1}{2}$?

8. Mary, had $\frac{1}{5}$ of a pie. She $\frac{1}{3}$ of it to Hanah. What part of a pie did Hannah receive?

Solution—If Mary, had $\frac{1}{5}$ of a pie, gave $\frac{1}{3}$ of it to Hannah, Hannah received $\frac{1}{3}$ of $\frac{1}{5}$ of a pie; $\frac{1}{3}$ of $\frac{1}{5}$ is one of the three equal parts into which $\frac{1}{5}$ maybe divided; if each fifth is divided into 3 equal parts, $\frac{3}{5}$ will be divided into 5 times 3, or 15 equal parts; hence each part is $\frac{1}{15}$ of the pie.

9. Philip, had $\frac{1}{4}$ of an orange, gave $\frac{1}{5}$ of it to Peter; what part of an orange did Peter receive?

10. A had $\frac{1}{3}$ of a dollar, and gave $\frac{1}{5}$ of it to B; what part of a dollar did B receive?

11. Since $\frac{1}{3}$ of $\frac{1}{4}$ equals $\frac{1}{12}$, how may the same result is obtained without the analysis?

Solution—By multiplying the denominators, 3 and 4, together for the denominator of the result.

12. Since $\frac{1}{4}$ of $\frac{1}{5}$ equals $\frac{1}{20}$, how may we obtain the same result without the analysis?

13. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{1}{6}$? $\frac{1}{5}$ of 1? $\frac{1}{5}$ of $\frac{1}{7}$?

14. What is $\frac{1}{3}$ of $\frac{1}{6}$? $\frac{1}{5}$ of $\frac{1}{8}$? $\frac{1}{7}$ of $\frac{1}{3}$? $\frac{1}{8}$ pf $\frac{1}{9}$?

15. What is $\frac{1}{6}$ of $\frac{1}{4}$? $\frac{1}{7}$ of $\frac{1}{7}$? $\frac{1}{9}$ of $\frac{1}{5}$? $\frac{1}{10}$ of $\frac{1}{11}$?

16. A man, owned $\frac{1}{8}$ of a farm, sold $\frac{1}{6}$ of it to his neighbor; what part of the farm did he sell?

17. Susan bought $\frac{1}{6}$ of a cake, and gave Eliza $\frac{1}{3}$ of it; how much did Eliza receive?

18. Mary is 8 years old, and her age is $\frac{1}{2}$ of $\frac{1}{3}$ of her mother's age; how old is her mother?

19. A man had $\frac{1}{7}$ of the stock of a bank, sold $\frac{1}{4}$ of it; what part of his stock did he keep?

20. Carlo, found $\frac{1}{5}$ of a pound of peanuts, let Tim take $\frac{1}{4}$ of it; what part of a pound did Carlo keep?

21. A stamp cost 12 cents, and $\frac{2}{3}$ of its cost is $\frac{1}{3}$ of $\frac{1}{4}$: of the cost of the stamp; what is the cost of the stamp.

22. What is $\frac{2}{3}$ of $\frac{4}{5}$?

Solution— $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$ and if $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$, $\frac{1}{3}$ of $\frac{4}{5}$ is 4 times $\frac{1}{15}$ which are $\frac{4}{15}$, and $\frac{2}{3}$ of $\frac{4}{5}$ are 2 times $\frac{4}{15}$, or $\frac{8}{15}$.

23. What is $\frac{1}{3}$ of $\frac{2}{3}$? $\frac{1}{4}$ of $\frac{3}{5}$? $\frac{1}{5}$ of $\frac{4}{6}$?

24. What is $\frac{1}{4}$ of $\frac{5}{7}$? $\frac{1}{6}$ of $\frac{5}{8}$? $\frac{1}{7}$ of $\frac{5}{6}$?

25. What is $\frac{1}{5}$ of $\frac{6}{7}$? $\frac{1}{8}$ of $\frac{7}{9}$? $\frac{1}{6}$ of $\frac{4}{12}$?

26. What is $\frac{2}{3}$ of $\frac{5}{6}$? $\frac{3}{4}$ of $\frac{5}{6}$? $\frac{4}{5}$ of $\frac{6}{8}$?

27. What is $\frac{3}{5}$ of $\frac{4}{6}$? $\frac{5}{6}$ of $\frac{5}{7}$? $\frac{3}{4}$ of $\frac{3}{6}$?

28. What is $\frac{2}{5}$ of $\frac{4}{7}$? $\frac{7}{10}$ of $\frac{5}{8}$? $\frac{5}{9}$ of $\frac{7}{10}$?

29. Since $\frac{2}{3}$ of $\frac{4}{5}$ equals $\frac{8}{15}$, in what manner may we obtain the same result by omitting the analysis?

30. How, do we find a fractional part of a fraction without going through with the analysis?

31. What is $\frac{3}{8}$ of $\frac{3}{9}$? $\frac{2}{5}$ of $\frac{2}{7}$? $\frac{3}{4}$ of $\frac{3}{5}$?

32. What is $\frac{5}{7}$ of $\frac{2}{3}$? $\frac{6}{8}$ of $\frac{3}{4}$? $\frac{7}{6}$ of $\frac{3}{7}$?

33. What is $\frac{3}{4}$ of $\frac{8}{6}$? $\frac{4}{5}$ of $\frac{10}{8}$? $\frac{3}{7}$ of $\frac{14}{9}$?

34. A boy had $\frac{3}{4}$ of a dollar, and gave away $\frac{2}{5}$ of it how much did he give away?

35. A man had $\frac{2}{5}$ of the stock of a bank, sold $\frac{3}{8}$ of it; how much stock did he sell?

36. Bowman had $\frac{2}{7}$ of a melon, gave $\frac{2}{3}$ of it to Martin; what part of $\frac{2}{7}$ remained?

37. I had $\frac{3}{4}$ of a kg of apples, I gave $\frac{3}{4}$ of them to Westlake; what part of a kg is left?

38. Eva received $\frac{4}{5}$ of her father's money, and spent $\frac{5}{6}$ of it; how much is left?

39. I lost $\frac{1}{4}$ of my money, I found $\frac{1}{2}$ of what I lost, and then realized I now have \$70; how much had I at first?

40. Harding lost $\frac{8}{9}$ of his money, and then found as much as $\frac{1}{6}$ of the remainder; what part of his money did he then have?

41. Annie had $\frac{2}{5}$ of a pound of candies, shared them equally with 5 of her schoolmates; what part of a pound did each receive?

42. A squirrel fell $\frac{5}{6}$ of the distance from the top of a tree to the ground, and then climbed $\frac{1}{4}$ of the distance he was from the ground; what part of the whole distance is he from the ground?

Answers

WHAT is $\frac{1}{3}$ of $\frac{1}{4}$?

Solution— $\frac{1}{3}$ of $\frac{1}{4}$ is one of the three equal parts into which $\frac{1}{4}$ may be divided; if each fourth is divided into three equal parts, 4 fourths or the unit will be divided into 4 times 3, or 12 equal parts; hence, each part is $\frac{1}{12}$ of a unit. Therefore, $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$.

2. What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{8}$? $\frac{1}{2}$ of $\frac{1}{10}$?

$\frac{1}{8}$; $\frac{1}{16}$; $\frac{1}{20}$.

3. What is $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{6}$?

$\frac{1}{15}$; $\frac{1}{6}$; $\frac{1}{18}$.

4. What is $\frac{1}{4}$ of $\frac{1}{6}$? $\frac{1}{6}$ of $\frac{1}{5}$? $\frac{1}{7}$ of $\frac{1}{6}$?

$\frac{1}{24}$; $\frac{1}{30}$; $\frac{1}{42}$.

5. What is $\frac{1}{5}$ of $\frac{1}{3}$? $\frac{1}{7}$ of $\frac{1}{2}$? $\frac{1}{7}$ of $\frac{1}{3}$?

$\frac{1}{15}$; $\frac{1}{14}$; $\frac{1}{21}$.

6. What is $\frac{1}{4}$ of $\frac{1}{8}$? $\frac{1}{6}$ of $\frac{1}{8}$? $\frac{1}{7}$ of $\frac{1}{9}$?

$\frac{1}{32}$; $\frac{1}{48}$; $\frac{1}{63}$.

7. What is $\frac{1}{6}$ of $\frac{1}{12}$? $\frac{1}{3}$ of $\frac{1}{12}$? $\frac{1}{35}$ of $\frac{1}{2}$?

$\frac{1}{72}$; $\frac{1}{36}$; $\frac{1}{70}$.

8. Mary, had $\frac{1}{5}$ of a pie. She $\frac{1}{3}$ of it to Hanah. What part of a pie did Hannah receive?

Solution—If Mary, had $\frac{1}{5}$ of a pie, gave $\frac{1}{3}$ of it to Hannah, Hannah received $\frac{1}{3}$ of $\frac{1}{5}$ of a pie; $\frac{1}{3}$ of $\frac{1}{5}$ is one of the three equal parts into which $\frac{1}{5}$ maybe divided; if each fifth is divided into 3 equal parts, $\frac{1}{5}$ will be divided into 5 times 3, or 15 equal parts; hence each part is $\frac{1}{15}$ of the pie.

9. Philip, had $\frac{1}{4}$ of an orange, gave $\frac{1}{5}$ of it to Peter; what part of an orange did Peter receive?

Solving as in the last question, we find that Peter received $\frac{1}{20}$ of an orange.

10. A had $\frac{1}{3}$ of a dollar, and gave $\frac{1}{5}$ of it to B; what part of a dollar did B receive?

If A had $\frac{1}{3}$ of a dollar, and gave $\frac{1}{5}$ of it to B, B received $\frac{1}{5}$ of $\frac{1}{3}$, or $\frac{1}{15}$ of a dollar.

11. Since $\frac{1}{3}$ of $\frac{1}{4}$ equals $\frac{1}{12}$, how may the same result is obtained without the analysis?

Solution—By multiplying the denominators, 3 and 4, together for the denominator of the result.

12. Since $\frac{1}{4}$ of $\frac{1}{5}$ equals $\frac{1}{20}$, how may we obtain the same result without the analysis?

By multiplying the denominators. 4 and 5, together for the denominator of the result.

13. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{1}{6}$? $\frac{1}{5}$ of 1? $\frac{1}{5}$ of $\frac{1}{7}$?

$\frac{1}{6}$; $\frac{1}{15}$; $\frac{1}{30}$; $\frac{1}{35}$.

14. What is $\frac{1}{3}$ of $\frac{1}{6}$? $\frac{1}{5}$ of $\frac{1}{8}$? $\frac{1}{7}$ of $\frac{1}{3}$? $\frac{1}{8}$ pf $\frac{1}{9}$?

$\frac{1}{18}$; $\frac{1}{40}$; $\frac{1}{21}$; $\frac{1}{72}$.

15. What is $\frac{1}{6}$ of $\frac{1}{4}$? $\frac{1}{7}$ of $\frac{1}{7}$? $\frac{1}{9}$ of $\frac{1}{5}$? $\frac{1}{10}$ of $\frac{1}{11}$?

$\frac{1}{24}$; $\frac{1}{49}$; $\frac{1}{45}$; $\frac{1}{10}$.

16. A man, owned $\frac{1}{8}$ of a farm, sold $\frac{1}{6}$ of it to his neighbor; what part of the farm did he sell?

If he sold $\frac{1}{6}$ to his neighbor, he sold $\frac{1}{6}$ of $\frac{1}{8}$ of the whole farm; multiplying the denominators together, we have 48; therefore $\frac{1}{6}$ of $\frac{1}{8}$ is $\frac{1}{48}$.

17. Susan bought $\frac{1}{6}$ of a cake, and gave Eliza $\frac{1}{3}$ of it; how much did Eliza receive?

If Susan gave Eliza $\frac{1}{3}$, she must have given her $\frac{1}{3}$ of $\frac{1}{6}$ of the cake, and multiplying the denominators together, we have 18; therefore $\frac{1}{3}$ of $\frac{1}{6}$ is $\frac{1}{18}$.

18. Mary is 8 years old, and her age is $\frac{1}{2}$ of $\frac{1}{3}$ of her mother's age; how old is her mother?

$\frac{1}{2}$ of $\frac{1}{3}$ of her mother's age is $\frac{1}{6}$ of her mother's age if 8 years is $\frac{1}{6}$ of her mother's age, $\frac{6}{6}$ is 6 times 8, or 48 years, her mother's age.

19. A man had $\frac{1}{7}$ of the stock of a bank, sold $\frac{1}{4}$ of it; what part of his stock did he keep?

If he sold $\frac{1}{4}$, he retained $\frac{4}{4} - \frac{1}{4}$, or $\frac{3}{4}$ of his stock.

20. Carlo, found $\frac{1}{5}$ of a pound of peanuts, let Tim take $\frac{1}{4}$ of it; what part of a pound did Carlo keep?

If Tim took $\frac{1}{4}$, Carlo took $\frac{4}{4} - \frac{1}{4}$, or $\frac{3}{4}$; $\frac{1}{4}$ of $\frac{1}{5}$ is $\frac{1}{20}$, and $\frac{3}{4}$ is $\frac{3}{20}$, the part of a pound which Carlo kept.

21. A stamp cost 12 cents, and $\frac{2}{3}$ of its cost is $\frac{1}{3}$ of $\frac{1}{4}$: of the cost of the stamp; what is the cost of the stamp.

$\frac{2}{3}$ of 12 cents is 8 cents; $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$; if $\frac{1}{12}$ of the cost of the stamp is 8 cents, $\frac{12}{12}$ is 12 times 8 cents, or 96 cents.

22. What is $\frac{2}{3}$ of $\frac{4}{5}$?

Solution— $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$ and if $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$, $\frac{1}{3}$ of $\frac{4}{5}$ is 4 times $\frac{1}{15}$ which are $\frac{4}{15}$, and $\frac{2}{3}$ of $\frac{4}{5}$ are 2 times $\frac{4}{15}$, or $\frac{8}{15}$.

23. What is $\frac{1}{3}$ of $\frac{2}{3}$? $\frac{1}{4}$ of $\frac{3}{5}$? $\frac{1}{5}$ of $\frac{4}{6}$?

$\frac{2}{9}$; $\frac{3}{20}$; $\frac{2}{15}$.

24. What is $\frac{1}{4}$ of $\frac{5}{7}$? $\frac{1}{6}$ of $\frac{5}{8}$? $\frac{1}{7}$ of $\frac{5}{6}$?

$\frac{5}{28}$; $\frac{5}{48}$; $\frac{5}{42}$.

25. What is $\frac{1}{5}$ of $\frac{6}{7}$? $\frac{1}{8}$ of $\frac{7}{9}$? $\frac{1}{6}$ of $\frac{4}{12}$?

$\frac{6}{35}$; $\frac{7}{42}$; $\frac{1}{15}$.

26. What is $\frac{2}{3}$ of $\frac{5}{6}$? $\frac{3}{4}$ of $\frac{5}{6}$? $\frac{4}{5}$ of $\frac{6}{8}$?

$\frac{5}{9}$; $\frac{5}{8}$; $\frac{3}{5}$.

27. What is $\frac{3}{5}$ of $\frac{4}{6}$? $\frac{5}{6}$ of $\frac{5}{7}$? $\frac{3}{4}$ of $\frac{3}{6}$?

$\frac{2}{5}$; $\frac{25}{42}$; $\frac{3}{8}$.

28. What is $\frac{2}{5}$ of $\frac{4}{7}$? $\frac{7}{10}$ of $\frac{5}{8}$? $\frac{5}{9}$ of $\frac{7}{10}$?

$\frac{8}{35}$; $\frac{7}{16}$; $\frac{7}{18}$.

29. Since $\frac{2}{3}$ of $\frac{4}{5}$ equals $\frac{8}{15}$, in what manner may we obtain the same result by omitting the analysis?

By multiplying 2 by 4 for the numerator, and 3 by 5 for the denominator, of the result.

30. How, do we find a fractional part of a fraction without going through with the analysis?

By multiplying the numerators together for the numerator of the result, and the denominators for the denominator.

31. What is $\frac{3}{8}$ of $\frac{3}{9}$? $\frac{2}{5}$ of $\frac{2}{7}$? $\frac{3}{4}$ of $\frac{3}{5}$?

$\frac{1}{8}$; $\frac{4}{35}$; $\frac{9}{20}$.

32. What is $\frac{5}{7}$ of $\frac{2}{3}$? $\frac{6}{8}$ of $\frac{3}{4}$? $\frac{7}{6}$ of $\frac{3}{7}$?

$\frac{10}{21}$; $\frac{9}{16}$; $\frac{1}{2}$.

33. What is $\frac{3}{4}$ of $\frac{8}{6}$? $\frac{4}{5}$ of $\frac{10}{8}$? $\frac{3}{7}$ of $\frac{14}{9}$?

1; 1; $\frac{2}{3}$.

34. A boy had $\frac{3}{4}$ of a dollar, and gave away $\frac{2}{5}$ of it how much did he give away?

$\frac{3}{10}$.

35. A man had $\frac{2}{5}$ of the stock of a bank, sold $\frac{3}{8}$ of it; how much stock did he sell?

He sold $\frac{3}{8}$ of $\frac{2}{5}$; 3 times 2 are 6, and 8 times 5 are 40; therefore he sold $\frac{6}{40}$, or $\frac{3}{20}$ of the stock.

36. Bowman had $\frac{2}{7}$ of a melon, gave $\frac{2}{3}$ of it to Martin; what part of $\frac{2}{7}$ remained?

If he gave away $\frac{2}{3}$ of $\frac{2}{7}$, he had remaining $\frac{3}{3} - \frac{2}{3}$, or $\frac{1}{3}$ of $\frac{2}{7}$.

37. I had $\frac{3}{4}$ of a kg of apples, I gave $\frac{3}{4}$ of them to Westlake; what part of a kg is left?

$\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$; $\frac{1}{4}$ of $\frac{3}{4}$ of a bushel is $\frac{3}{16}$ of a bushel.

38. Eva received $\frac{4}{5}$ of her father's money, and spent $\frac{5}{6}$ of it; how much is left?

$\frac{6}{6} - \frac{5}{6} = \frac{1}{6}$, what remained; $\frac{1}{6}$ of $\frac{4}{5} = \frac{4}{30}$, or $\frac{2}{15}$ of the money.

39. I lost $\frac{1}{4}$ of my money, I found $\frac{1}{2}$ of what I lost, and then realized I now have \$70; how much had I at first?

$\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$, what remained; I found $\frac{1}{2}$ of $\frac{1}{4}$, or $\frac{1}{8}$; $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$; if $\frac{7}{8}$ of my money = \$70, $\frac{8}{8}$ is \$80, this is what I had at first.

40. Harding lost $\frac{8}{9}$ of his money, and then found as much as $\frac{1}{6}$ of the remainder; what part of his money

lid he then have?

$\frac{9}{9} - \frac{8}{9} = \frac{1}{9}$, what remained; he found $\frac{1}{6}$ of $\frac{1}{8}$, or $\frac{1}{5}4$ and then had $\frac{1}{9} + \frac{1}{5}4$, or $\frac{7}{5}4$ of his money.

41. Annie had $\frac{2}{5}$ of a pound of candies, shared them equally with 5 of her schoolmates; what part of a pound did each receive?

If she shared them with 5 schoolmates, there were 6 among whom to divide them, and each received $\frac{1}{6}$ of $\frac{2}{5}$, or $\frac{1}{15}$ of a pound.

42. A squirrel fell $\frac{5}{6}$ of the distance from the top of a tree to the ground, and then climbed $\frac{1}{4}$ of the distance he was from the ground; what part of the whole distance is he from the ground?

$\frac{1}{6}$ of the whole distance equals the distance he was from the ground; he ascended $\frac{1}{4}$ of $\frac{1}{6}$, or $\frac{1}{24}$ of the whole distance; $\frac{1}{6} + \frac{1}{24}$, or $\frac{5}{24}$ of the whole distance, equals the distance he was from the ground.

2.10 Lesson 10 - *Multiplication of Fractions*

HOW many are 4 times $\frac{3}{8}$?

Solution—4 times $\frac{3}{8}$ is $1\frac{6}{8}$, which equals $\frac{3}{2}$, or $1\frac{1}{2}$.

2. How many are 3 times $\frac{3}{6}$? 3 times $\frac{2}{9}$?
3. How many are 4 times $\frac{6}{8}$? 3 times $\frac{2}{6}$?
4. How many are 7 times $\frac{3}{14}$? 5 times $\frac{4}{10}$?
5. How many are 4 times $\frac{6}{12}$? 6 times $\frac{7}{12}$?
6. How many are 3 times $\frac{6}{9}$? 4 times $\frac{2}{6}$?
7. How many are 8 times $\frac{2}{4}$? 8 times $\frac{3}{6}$?
8. How many are 5 times $\frac{3}{6}$? 5 times $\frac{7}{6}$?
9. How many are 3 times $\frac{7}{9}$?

Solution—3 times $\frac{1}{9}$ equals $\frac{3}{9}$, or $\frac{1}{3}$; if 3 times $\frac{1}{9}$ equals $\frac{1}{3}$, 3 times $\frac{7}{9}$ equals 7 times $\frac{1}{3}$, or $\frac{7}{3}$; hence 3 times $\frac{7}{9}$ equals $\frac{7}{3}$.

How many are

10. 4 times $\frac{3}{8}$?

Solution—4 times $\frac{1}{8}$ equals $\frac{4}{8}$, or $\frac{1}{2}$; if 4 times $\frac{1}{8}$ equals $\frac{1}{2}$, 4 times $\frac{3}{8}$ equals 3 times $\frac{1}{2}$, or $\frac{3}{2}$; hence 4 times $\frac{3}{8} = \frac{3}{2}$.

11. 3 times $\frac{8}{9}$?
12. 5 times $\frac{11}{15}$?
13. 5 times $\frac{9}{10}$?
14. 6 times $\frac{18}{24}$?
15. 7 times $\frac{18}{21}$?
16. 6 times $\frac{11}{18}$?
17. 7 times $\frac{12}{21}$?
18. 8 times $\frac{12}{24}$?
19. 9 times $\frac{24}{27}$?

20. Since 3 times $\frac{7}{9}$ equals $\frac{7}{3}$, how may this result be obtained by omitting the analysis?

21. How, then, may a fraction be multiplied by a number which will divide the denominator?

22. How do we multiply a fraction by a number that will not divide the denominator?

23. What is the effect of dividing the denominator of a fraction by a number?

How many are

24. 4 times $\frac{5}{8}$?
25. 6 times $\frac{5}{6}$?
26. 7 times $\frac{3}{14}$?
27. 5 times $1\frac{9}{10}$?

4.3 Lesson 3 - *To find the base*

THOMAS sold his guitar case \$25, and thereby gained 25 percent; what was the cost of the case?

Solution—If he gained 25 percent then $\frac{25}{100}$, or $\frac{1}{4}$, of the cost equals the gain, which added to $\frac{4}{4}$, the cost, is $\frac{5}{4}$ of the cost, which equals \$25. If $\frac{5}{4}$ of the cost equals \$25, $\frac{1}{4}$ of the cost equals $\frac{1}{5}$ of \$25, which is \$5, and $\frac{4}{4}$, or the cost, equals 4 times \$5, which are \$20.

2. Mary sold her collectible dolls for \$14, which was at a gain of 40 percent; what did these dolls cost?
3. A farmer sold a cow for \$230, and thereby gained 15 percent what is the value of the cow?
4. A student sold his collection of comic books for \$140, and thereby lost 30 percent; what was the cost of the books?
5. By selling a CD for \$8, Mary lost 20 percent; what was the value of the CD?
6. A used computer was bought for \$150, and then sold at a gain of 20 percent; for what was the price at which it was sold?
7. A shopkeeper gained 20 percent by selling cloth at \$6 per meter; at what price should he have sold it to gain 25 percent?
8. If by selling land at \$7500 an acre I gain 25 percent how must I sell it to lose 40 percent?
9. If a manufacturer sells computer chips at 39 cents a set, and thereby gains 30 percent for what shall he sell it to lose 40 percent?
10. A boat was sold for \$9100, which was at a loss of 35 percent; for how much should it have been sold to gain 40 percent?
11. Taylor lost 60 percent on a stock by selling it for \$40 per share; what should he have received for his gain to be 60 percent?

Solution—If he lost 60 percent he sold it for 40 percent.; hence 40 percent or $\frac{2}{5}$ of the cost, equals \$40, and the cost was \$100; to gain 60 percent it must be sold for $\frac{8}{5}$ of the cost, or \$160.

12. Jamal sold his house for \$240,000, and lost 4 percent; what would he have gained percent by selling it for \$300,000?
 13. A car was sold for \$9000, which was 10 percent less than its value; what would have been the gain percent if it had been sold for \$12000?
 14. Mr. Bowman sold 2 old books for \$15 each; on one he gained 25 percent and on the other he lost 25 percent; how much did he lose by the transaction?
- Solution**—On the first $\frac{25}{100}$, or $\frac{1}{4}$ of the cost, equals the gain which added to the cost equals $\frac{5}{4}$ of the cost, which is \$15; $\frac{1}{4}$ of the cost or the gain equals $\frac{1}{5}$ of \$15, or \$3. On the second book, $\frac{25}{100}$ or $\frac{1}{4}$ of the cost, equals the loss, which subtracted from the cost, equals $\frac{1}{3}$ of the cost, or \$15, of \$5.. Since he gained \$3 on the first books and lost \$5 on the second, his loss is \$2.
15. A tailor sold 2 coats for \$120 each; on one he gained 20 percent and on the other he lost 20 percent; did he gain or lose by the sale, and how much?
- Solution**—If he gained 20 percent on the first, then $\frac{6}{5}$ of the cost equals \$12, and $\frac{1}{5}$ of the cost, or the gain, equals \$2; if he lost 20 percent on the second, then $\frac{4}{5}$ of the cost equals \$12, and $\frac{1}{5}$ of the cost, or the loss, equals \$3; since he lost \$3 on one and gained \$2 on the other, he lost \$1.
16. B bought a TV for \$420, which was 40 percent less than its value; he sold it for 30 percent more than its value; what was the gain?
 17. A man sold 2 bikes for \$80 each; on one he lost 20 percent and on the other he gained 25 percent How much was gained or lost by the transaction?
 18. A merchant sold a microwave oven for \$300, and thereby lost 25 percent; he then bought another for \$300, and this time sold it at for a gain of 25 percent; what was his total gain or loss?
 19. Martha sold a painting so that $\frac{2}{3}$ of what she received for it equaled $\frac{4}{5}$ of the cost; did she gain or lose? and what percent?
 20. Terry sold stock A and stock B for \$1200, receiving 5 times as much for the stock A as for B; on stock A he gained 25 percent and on stock B he lost 20 percent; what was the total gain?

Answers

THOMAS sold his guitar case \$25, and thereby gained 25 percent; what was the cost of the case?

Solution—If he gained 25 percent then $\frac{25}{100}$, or $\frac{1}{4}$, of the cost equals the gain, which added to $\frac{4}{4}$, the cost, is $\frac{5}{4}$ of the cost, which equals \$25. If $\frac{5}{4}$ of the cost equals \$25, $\frac{1}{4}$ of the cost equals $\frac{1}{5}$ of \$25, which is \$5, and $\frac{4}{4}$, or the cost, equals 4 times \$5, which are \$20.

2. Mary sold her collectible dolls for \$14, which was at a gain of 40 percent; what did these dolls cost?
\$10.
3. A farmer sold a cow for \$230, and thereby gained 15 percent what is the value of the cow?
\$200.
4. A student sold his collection of comic books for \$140, and thereby lost 30 percent; what was the cost of the books?
\$200.
5. By selling a CD for \$8, Mary lost 20 percent; what was the value of the CD?
\$10.
6. A used computer was bought for \$150, and then sold at a gain of 20 percent; for what was the price at which it was sold?
\$180.
7. A shopkeeper gained 20 percent by selling cloth at \$6 per meter; at what price should he have sold it to gain 25 percent?
 $\$6\frac{1}{4}$.
8. If by selling land at \$7500 an acre I gain 25 percent how must I sell it to lose 40 percent?
\$3600.
9. If a manufacturer sells computer chips at 39 cents a set, and thereby gains 30 percent for what shall he sell it to lose 40 percent?
18 cents.
10. A boat was sold for \$9100, which was at a loss of 35 percent; for how much should it have been sold to gain 40 percent?
\$19600.
11. Taylor lost 60 percent on a stock by selling it for \$40 per share; what should he have received for his gain to be 60 percent?
Solution—If he lost 60 percent he sold it for 40 percent.; hence 40 percent or $\frac{2}{5}$ of the cost, equals \$40, and the cost was \$100; to gain 60 percent it must be sold for $\frac{8}{5}$ of the cost, or \$160.
12. Jamal sold his house for \$240,000, and lost 4 percent; what would he have gained percent by selling it for \$300,000?
20%
13. A car was sold for \$9000, which was 10 percent less than its value; what would have been the gain percent if it had been sold for \$12000?
12%
14. Mr. Bowman sold 2 old books for \$15 each; on one he gained 25 percent and on the other he lost 25 percent; how much did he lose by the transaction?
Solution—On the first $\frac{25}{100}$, or $\frac{1}{4}$ of the cost, equals the gain which added to the cost equals $\frac{5}{4}$ of the cost, which is \$15; $\frac{1}{4}$ of the cost or the gain equals $\frac{1}{5}$ of \$15, or \$3. On the second book, $\frac{25}{100}$ or $\frac{1}{4}$ of the cost, equals the loss, which subtracted from the cost, equals $\frac{1}{3}$ of the cost, or \$15, of \$5.. Since he gained \$3 on the first books and lost \$5 on the second, his loss is \$2.
15. A tailor sold 2 coats for \$120 each; on one he gained 20 percent and on the other he lost 20 percent; did he gain or lose by the sale, and how much?
Solution—If he gained 20 percent on the first, then $\frac{6}{5}$ of the cost equals \$12, and $\frac{1}{5}$ of the cost, or the gain, equals \$2; if he lost 20 percent on the second, then $\frac{4}{5}$ of the cost equals \$12, and $\frac{1}{5}$ of the cost, or the

loss, equals \$3; since he lost \$3 on one and gained \$2 on the other, he lost \$1.

16. B bought a TV for \$420, which was 40 percent less than its value; he sold it for 30 percent more than its value; what was the gain?

\$420 is 60 percent or $\frac{3}{5}$ of the value; hence the value was \$700; he sold it for 130 percent or $\frac{13}{10}$ of its value, which is \$910; he gained \$910 - \$420 = \$490.

17. A man sold 2 bikes for \$80 each; on one he lost 20 percent and on the other he gained 25 percent. How much was gained or lost by the transaction?

If he lost 20 percent on the first, $\frac{4}{5}$ of the cost equals \$80, and $\frac{1}{5}$, or the loss equals \$20; if he gained 25 percent on the second, $\frac{5}{4}$ of the cost equals \$80, and $\frac{1}{4}$ of the cost, or the gain, equals \$16; he therefore lost \$4.

18. A merchant sold a microwave oven for \$300, and thereby lost 25 percent; he then bought another for \$300, and this time sold it at for a gain of 25 percent; what was his total gain or loss?

\$300 equals $\frac{3}{4}$ of the cost of the first stove, and $\frac{1}{4}$ of the cost, or the loss, is \$10; $\frac{1}{4}$ of \$30, or $\$7\frac{1}{2}$, is the gain on the second stove; hence the loss on the transaction was $\$2\frac{1}{2}$.

19. Martha sold a painting so that $\frac{2}{3}$ of what she received for it equaled $\frac{4}{5}$ of the cost; did she gain or lose? and what percent?

We find $\frac{3}{3}$ of what she received for the painting equals $\frac{6}{5}$ of the cost; hence she gained $\frac{1}{5}$ of the cost, or 20 percent.

20. Terry sold stock A and stock B for \$1200, receiving 5 times as much for the stock A as for B; on stock A he gained 25 percent and on stock B he lost 20 percent; what was the total gain?

\$1200 is 6 times the value of the chain, hence the Watch brought \$100, and the chain \$20; \$100 is $\frac{5}{4}$ of the cost of the watch, and $\frac{1}{4}$, or the gain, is \$20; \$20 is $\frac{4}{5}$ of the cost of the chain, and $\frac{1}{5}$, or the loss, is \$5; hence he Rained \$20 - \$5, or \$15.

4.4 Lesson 4 - *To find the Base*

A man gained 25 percent by selling his stocks for \$20 more than it cost; what is the cost?

Solution—At a gain of 25 percent $\frac{25}{100}$, or $\frac{1}{4}$, of the cost equals the gain, which is \$20; if $\frac{1}{4}$ of the cost equals \$20, $\frac{4}{4}$ of the cost equals 4 times \$20, or \$80.

2. An investor gained 30 percent by selling some shares for \$9 more than the cost; what did the shares cost?

3. A, by selling his music player for \$60 less than it cost, lost 15 percent; what is the cost of the player?

4. A machine was sold for 2000 dollars less than cost, which was at a loss of 40 percent; what is its cost?

5. Four is 10 percent 5 is 20 percent and 6 is 25 percent of what numbers?

6. Eight is 40 percent 9 is 30 percent and 12 is 12 percent of what numbers?

7. Thirty is 25 cent, less, and 25 percent more, than what numbers?

8. A man gained \$2000 by selling a boat for 20 percent more than its value; what would he have gained by selling it for 10 percent above its value?

9. A piano was sold for \$600 less than its value, which was at a loss of 30 percent; what would have been the gain percent if it had been sold for \$2500?

10. \$24 is 4 percent of the sum of A's and B's money; how much money has each, provided A has twice as much as B?

11. A man spends \$120 to purchase some goods. The sales tax is 10%. How much was the cost of the goods?

2. A man receives 25 percent commission for finding a suitable land for construction; how many dollars' worth land can be purchased for \$20,000, after taking out his commission?

13. An agent is given \$216 to buy goods, and is to keep 8 percent of the money spent; what is the amount of money spent?

14. Frank received \$28000 to invest in land after deducting his commission, which is 12 percent on the amount invested; what is his commission?
15. A's shop is valued at \$90,000; for what sum must he have it insured, at 10 percent cost, so that in case of loss he may receive both the value of the shop and the insurance premium he paid?
16. At 2 percent insurance rate, what must a house worth \$490,000 be insured for, so that the premium maybe included in the amount received in case of loss?
17. How many meters of cloth, at \$4 a meter, should a merchant buy that by selling it at a profit of 20 percent he can gain \$800?
18. A contractor receives \$5300 to purchase boards and roofing material. What sum will he spend for each, after deducting his commission, which is 6 percent of the money spent, provided he spends 4 times as much for boards as for roofing materials?

Answers

A man gained 25 percent by selling his stocks for \$20 more than it cost; what is the cost?

Solution—At a gain of 25 percent $\frac{25}{100}$, or $\frac{1}{4}$, of the cost equals the gain, which is \$20; if $\frac{1}{4}$ of the cost equals \$20, $\frac{4}{4}$ of the cost equals 4 times \$20, or \$80.

2. An investor gained 30 percent by selling some shares for \$9 more than the cost; what did the shares cost?
\$30.
3. A, by selling his music player for \$60 less than it cost, lost 15 percent; what is the cost of the player?
\$40.
4. A machine was sold for 2000 dollars less than cost, which was at a loss of 40 percent; what is its cost?
5000 dollars.
5. Four is 10 percent 5 is 20 percent and 6 is 25 percent of what numbers?
40; 25; 24.
6. Eight is 40 percent 9 is 30 percent and 12 is 12 percent of what numbers?
20; 80; 100.
7. Thirty is 25 cent, less, and 25 percent more, than what numbers?
40; 24.
8. A man gained \$2000 by selling a boat for 20 percent more than its value; what would he have gained by selling it for 10 percent above its value?
If he gained \$20 by selling for 20 percent above its value, by selling for 10 percent above its value, which is $\frac{1}{2}$ of 20 percent he would have gained $\frac{1}{2}$ of \$20, or \$10.
9. A piano was sold for \$600 less than its value, which was at a loss of 30 percent; what would have been the gain percent if it had been sold for \$2500?
Since \$600 was 30 percent or $\frac{3}{10}$ of the value, the value was \$2000; \$500 is $\frac{1}{4}$, or 25 percent of the value.
10. \$24 is 4 percent of the sum of A's and B's money; how much money has each, provided A has twice as much as B?
The sum of their savings is \$600; of which A has $\frac{2}{3}$, or \$400; B, $\frac{1}{3}$, or \$200.
11. A man spends \$120 to purchase some goods. The sales tax is 10%. How much was the cost of the goods?
 $\frac{1}{5}$, his sales tax, + $\frac{5}{5}$, what he expended, = $\frac{6}{5}$ of what he expended, which is \$120; hence his sales tax was \$20.
12. A man receives 25 percent commission for finding a suitable land for construction; how many dollars' worth land can be purchased for \$20,000, after taking out his commission?
 $\frac{5}{4}$ of what he expended equals \$20,000; hence $\frac{4}{5}$ =16000.
13. An agent is given \$216 to buy goods, and is to keep 8 percent of the money spent; what is the amount of money spent?

As often as he paid 2 dollars for 1 toy he paid 8 dollars for 2 balls, hence 1 toy and 2 balls cost 10 dollars, and 3 were sold for 9 dollars; hence he lost on 1 toys and 2 balls 1 dollars, and to lose 10 dollars he must sell 10 times 1 toys and 2 balls, or 10 toys and 20 balls.

6.5 Lesson 5 - Age and Step Problems

A's AGE equals 4 times B's, but in 5 years A's age will be only 3 times B's; how old is each?

Solution—By the first condition, 4 times B's age equals A's age, hence, the difference of their ages is 3 times B's age, and once B's age equals $\frac{1}{3}$ of the difference of their ages; in 5 years, 3 times B's age equals A's age, hence, twice B's age equals the difference of their ages, and once B's age then equals $\frac{1}{2}$ of the difference. Therefore, 5 years is the difference between and $\frac{1}{3}$ of the difference of their ages, or $\frac{1}{6}$ of the difference of their ages; and $\frac{6}{6}$, or the difference of their ages, is 6 times 5, or 30 years. If 3 times B's age equals 30 years, B's age is $\frac{1}{3}$ of 30 years, or 10 years, and A's age is 4 times 10, or 40 years.

2. John is 5 times as old as Oliver, but in 8 years he will be only 3 times as old; what is the age of each?
3. Mary is $\frac{1}{4}$ as old as her aunt, but in 20 years she will be $\frac{1}{2}$ as old; what is the age of each?
4. Henry is $\frac{1}{5}$ as old as his father, but in 25 years he will be $\frac{3}{5}$ as old; what is the age of each?
5. Ten years ago, when I first met Mr. Morgan, I was $\frac{1}{4}$ as old as he, but now I am $\frac{1}{2}$ as old as he is; what is each of our ages?
6. Sixteen years ago, when Agnew married, he was times as old as his wife, but now he is only twice as old, what is the age of each?
7. A rabbit is 30 meters before a hound, and runs 3 meters while the hound runs 6; how many meters must the hound run to catch the rabbit?
8. Stephen is 40 steps ahead of James, and takes 5 steps to James's 7; how many steps must James take to catch Stephen, supposing their steps are equal?
9. A rabbit takes 2 leaps while a hound takes 1, but 1 of the hound's leaps equals 4 of the hare's; how much does the hound gain on the hare in taking one leap?
10. A rabbit is 30 leaps ahead of a hound, and takes 4 leaps while the hound takes 2, but 2 of the hound's leaps equal 8 of the rabbit's; how many leaps must the hound take to catch the rabbit?
11. A fox is 40 leaps ahead of a hound, and takes 3 leaps while the hound takes 2, but 2 of the hound's leaps equal 4 of the fox's; in how many leaps will the hound catch the fox?
12. A thief is 20 steps before an officer, and takes 6 steps while the officer takes 5, but 5 of the officer's steps equal 8 of the thief's; how far will the thief run before he is overtaken?
13. A rabbit is 60 leaps ahead of a hound, and takes 9 leaps while the hound takes 3, but 2 of the hound's leaps equal 7 of the rabbit's; how many leaps will the rabbit take before being caught?
14. Twenty-five years ago Willard was $\frac{1}{7}$ as old as his uncle, but 5 years ago he was $\frac{1}{3}$ as old; how old is each at present?
15. Four years ago B's house was four times as old as his barn, but 2 years from now it will be only twice as old; how long was each built?
16. Three years ago Emma's doll was only $\frac{1}{5}$ of the age of herself, but 7 years hence it will be $\frac{3}{5}$ of her age; what is the age of each?
17. B takes 30 steps to overtake C; how far was C ahead of B when they started, provided B takes 2 steps while C takes 3, and 2 of B's equal 5 of C's steps?
18. E takes 60 steps before he is overtaken by D; how many steps does D take to catch E, provided E takes 4 steps while D takes 3, and 5 of D's equal 8 of E's, and how far ahead was E when they started?
19. M and N are 60 meters apart, and approach each other; how far will each travel before they meet, provided M takes 3 steps while N takes 6, and 2 of M's equal 6 of N's steps?
20. A and B are 150 of B's steps apart, and approach each other; how many steps will each take before they are together, if 4 of A's steps equal 8 of B's, and B takes 9 steps while A takes 3?

Answers

A's AGE equals 4 times B's, but in 5 years A's age will be only 3 times B's; how old is each?

Solution—By the first condition, 4 times B's age equals A's age, hence, the difference of their ages is 3 times B's age, and once B's age equals $\frac{1}{3}$ of the difference of their ages; in 5 years, 3 times B's age equals A's age, hence, twice B's age equals the difference of their ages, and once B's age then equals $\frac{1}{2}$ of the difference. Therefore, 5 years is the difference between and $\frac{1}{3}$ of the difference of their ages, or $\frac{1}{6}$ of the difference of their ages; and $\frac{6}{6}$, or the difference of their ages, is 6 times 5, or 30 years. If 3 times B's age equals 30 years, B's age is $\frac{1}{3}$ of 30 years, or 10 years, and A's age is 4 times 10, or 40 years.

2. John is 5 times as old as Oliver, but in 8 years he will be only 3 times as old; what is the age of each?

The difference of their ages 4 times Oliver's, and once Oliver's = $\frac{1}{4}$ of the difference; in 8 years the difference will equal twice Oliver's, and once Oliver's will equal $\frac{1}{2}$ of the difference; therefore 8 years is the difference between $\frac{1}{2}$ of the difference and $\frac{1}{4}$ of the difference of their ages, or $\frac{1}{4}$ of the difference; hence the difference of their ages is 4 times 8, or 32 years, Oliver's age is 8 years, and John's 40.

3. Mary is $\frac{1}{4}$ as old as her aunt, but in 20 years she will be $\frac{1}{2}$ as old; what is the age of each?

The difference of their ages = 3 times Mary's, and once Mary's age = $\frac{1}{3}$ of the difference; in 20 years the difference will equal Mary's age; hence once the difference of their ages minus $\frac{1}{3}$ of the difference, or $\frac{2}{3}$ of the difference, equals 20 years; therefore the difference of their ages is 30 years, Mary's age is 10 years, and her aunt's 40.

4. Henry is $\frac{1}{5}$ as old as his father, but in 25 years he will be $\frac{3}{5}$ as old; what is the age of each.

The difference of their ages = 4 times Henry's age, and once Henry's age = $\frac{1}{4}$ of the difference; in 25 years the difference will equal $\frac{2}{3}$ of Henry's age, and once Henry's age will equal $\frac{3}{2}$ of the difference; hence $\frac{3}{2}$ of the difference minus $\frac{1}{4}$ of the difference, or $\frac{5}{4}$ of the difference, equals 25 years; therefore the difference is 20 years, Henry's age 5 years, and his father's 25.

5. Ten years ago, when I first met Mr. Morgan, I was $\frac{1}{4}$ as old as he, but now I am $\frac{1}{2}$ as old as he is; what is each of our ages.

The difference of our ages equaled 3 times my age, and my age equaled $\frac{1}{3}$ of the difference; in 10 years my age = once the difference; hence once the difference minus $\frac{1}{3}$ of the difference, or $\frac{2}{3}$ of the difference, is 10 years; therefore the difference is 15 years, my age is 15 years, and Morgan's 30.

6. Sixteen years ago, when Agnew married, he was times as old as his wife, but now he is only twice as old, what is the age of each?

The difference of their ages equaled twice the wife's age, and the wife's age equaled $\frac{1}{2}$ of the difference; in 16 years the wife's age = once the difference, hence once the difference minus $\frac{1}{2}$ of the difference, or $\frac{1}{2}$ of the difference, equals 16 years; therefore the difference is 32 years, the wife's age is 32 years, and Agnew's age is 64 years.

7. A rabbit is 30 meters before a hound, and runs 3 meters while the hound runs 6; how many meters must the hound run to catch the rabbit?

For every meter the hare runs the hound runs 2; hence to gain 1 meter he runs 2, and to gain 30 meters he must run 30 times 2, or 60 meters.

8. Stephen is 40 steps ahead of James, and takes 5 steps to James's 7; how many steps must James take to catch Stephen, supposing their steps are equal?

James takes 7 steps to gain 2, and to gain 40 he must take as many times 7 steps as 2 is contained times in 40, or 20; hence he must take 20 x 7, or 140 steps.

9. A rabbit takes 2 leaps while a hound takes 1, but 1 of the hound's leaps equals 4 of the hare's; how much does the hound gain on the hare in taking one leap?

Since 1 of the hound's leaps equals 4 of the hare's and he takes 1 leap while the hare takes 2, he gains 4 - 2, or 2 leaps.

10. A rabbit is 30 leaps ahead of a hound, and takes 4 leaps while the hound takes 2, but 2 of the hound's leaps equal 8 of the rabbit's; how many leaps must the hound take to catch the rabbit?

Since 2 of the hound's leaps equal 8 of the hare's, for the hare to run as fast as the hound it must take 8 leaps while the hound takes 2 leaps, but, by the problem, the hare takes only 4 while the hound takes 2, therefore the hound gains 8 - 4, or 4 of the hare's leaps in taking 2 leaps hence to gain 1 leap he must take $\frac{1}{4}$ of 2, or $\frac{1}{2}$ of a leap, and to gain 30 leaps, the distance the hare is ahead, he must take 30 times $\frac{1}{2}$ of a leap,

or 15 leaps.

11. A fox is 40 leaps ahead of a hound, and takes 3 leaps while the hound takes 2, but 2 of the hound's leaps equal 4 of the fox's; in how many leaps will the hound catch the fox?

We find the hound gains 1 leap on the fox in taking 2 leaps, and to gain 40 leaps he must take 40 times 2 leaps, or 80 leaps.

12. A thief is 20 steps before an officer, and takes 6 steps while the officer takes 5, but 5 of the officer's steps equal 8 of the thief's; how far will the thief run before he is overtaken?

We find that the thief loses 2 steps in taking 6 steps; hence to lose 20 steps, which are 10 times 2 steps, he must take 10 times 6 steps, or 60 steps.

13. A rabbit is 60 leaps ahead of a hound, and takes 9 leaps while the hound takes 3, but 2 of the hound's leaps equal 7 of the rabbit's; how many leaps will the rabbit take before being caught?

We find that the rabbit takes 6 leaps while the hound takes 2, but two of the hound's equal 7 of the rabbit's; hence the rabbit loses 1 leap in taking 6 leaps, and to lose 60 he will take 60 times 6, or 360 leaps.

14. Twenty-five years ago Willard was $\frac{1}{7}$ as old as his uncle, but 5 years ago he was $\frac{1}{3}$ as old; how old is each at present

The difference of their ages 25 years ago equaled 6 times Willard's, and Willard's age equaled $\frac{1}{6}$ of the difference; in 20 years, or 5 years ago, twice Willard's age = the difference, and his age = $\frac{1}{2}$ of the difference; hence $\frac{1}{2}$ of the difference minus 1 of the difference, or $\frac{1}{3}$ of the difference, equals 20 years; therefore the difference is 60 years, Willard's age is $30 + 5$, or 35 years, and his uncle's age is 95 years.

15. Four years ago B's house was four times as old as his barn, but 2 years from now it will be only twice as old; how long was each built?

The difference of age 4 years ago equaled 3 times the age of the barn, and the age of the barn equaled $\frac{1}{3}$ of the difference; 2 years hence once the difference will equal the age of the barn; hence once the difference minus $\frac{1}{3}$ of the difference, or $\frac{2}{3}$ of the difference, equals 6 years; therefore the difference is 9 years, the barn has been built 7 years, and the house 16.

16. Three years ago Emma's doll was only $\frac{1}{5}$ of the age of herself, but 7 years hence it will be $\frac{3}{5}$ of her age; what is the age of each?

The difference of age 3 years ago was 4 times the doll's age, and the doll's age equaled $\frac{1}{4}$ of the difference; 7 years hence the difference will equal of the doll's age, and the doll's age will be $\frac{3}{2}$ of the difference; $\frac{3}{2}$ of the difference minus $\frac{1}{4}$ of the difference, or $\frac{5}{4}$ of the difference, equals 10 years; therefore the difference is 8 years, the doll's age is $2 + 3$, or 5 years, and Emma's age is 13 years.

17. B takes 30 steps to overtake C; how far was C ahead of B when they started, provided B takes 2 steps while C takes 3, and 2 of B's equal 5 of C's steps?

We find that B in taking 2 steps gains 2 of C's steps; hence, in taking 30 steps he will gain 30 of C's steps, and therefore C was 30 steps ahead when they started.

18. E takes 60 steps before he is overtaken by D; how many steps does D take to catch E, provided E takes 4 steps while D takes 3, and 5 of D's equal 8 of E's, and how far ahead was E when they started?

Since D takes 3 steps while E takes 4, while E takes 60 steps D will take 45 steps; and since 5 of D's = 8 of E's, these 45 steps of D's = 72 of E's; hence E was ahead $72 - 60$, or 12 steps.

19. M and N are 60 meters apart, and approach each other; how far will each travel before they meet, provided M takes 3 steps while N takes 6, and 2 of M's equal 6 of N's steps?

Since 6 of N's equal 2 of M's, while M goes 3 steps N goes a distance equal to 2 of M's steps; hence the distances they travel are as 3 to 2; hence M goes $\frac{3}{5}$ of 60, or 36 rods, and N goes $\frac{2}{5}$ of 60, or 24 rods.

20. A and B are 150 of B's steps apart, and approach each other; how many steps will each take before they are together, if 4 of A's steps equal 8 of B's, and B takes 9 steps while A takes 3?

One of A's steps = 2 of B's steps; hence 3 of A's = 6 of B's steps; hence when B takes $\frac{9}{15}$ steps A goes a distance equal to 6 of B's steps; hence B takes $\frac{9}{15}$ of 150, or 90 steps, and A goes $\frac{6}{15}$ of 150, or 60 of B's steps, which equal 30 of A's steps.

and the other boat sails 5 times as fast going, and 10 times as fast returning; when do they meet?

19. Paid $\$34\frac{1}{2}$ for corn at $\$ \frac{3}{4}$, wheat at $\$1$, and oats at $\$ \frac{1}{2}$ a kg; sold $\frac{2}{3}$ of the corn and $\frac{1}{2}$ of the wheat for 50 percent advance, gaining on the corn $\frac{2}{3}$ as much, as on the wheat, and on the sale the cost of the oats; how much of each did I buy?

20. A person has two cups and a cover which weighs 30 ounces. If the first cup be covered it will weigh twice as much as the second, but if the second cup be covered it will weigh 3 times as much as the first; what is the weight of each cup?

21. A and B were engaged by a Pennsylvania farmer to dig 100 meters of ditch for $\$100$; and since the part which A was to dig was more difficult of excavation than that which B dug, it was agreed that A should receive 10 shillings per rod, and B 6 shillings per rod. They each received $\$50$ for their labor. How many meters did each dig?

Answers

A PERSON, when asked the hour of the day, replied that 2 hours ago the time past noon was $\frac{1}{3}$ of the time to midnight 2 hours hence; what is the time.

Solution—We find $\frac{4}{3}$ of the time to midnight 2 hours hence + 2 hours + 2 hours, equals 12 hours; hence, $\frac{4}{3}$ of the time to midnight, 2 hours hence, equals 8 hours, and $\frac{1}{3}$ of the time to midnight 2 hours hence, which was the time past noon 2 hours ago, equals $\frac{1}{4}$ of 8 hours, or 2 hours; hence the time now is 2 hours + 2 hours, or 4 hours past noon, or 4 o'clock P.M.

2. A man went to a store and spent 20 cents, and then, losing $\frac{3}{4}$ as much as remained, had $\frac{1}{3}$ as much as he had at first, minus $\$1$; how much did he have at first?

By the conditions, $\frac{1}{4}$ of what he had at first, - 5 cents = $\frac{1}{3}$ of what he had at first, - 100 cents; hence $\frac{1}{3}$ - $\frac{1}{4}$, or $\frac{1}{12}$ of what he had at first equals 100 - 5, or 95 cents; he had at first $\$11.40$.

3. A is 10 steps before B, and takes 2 steps while B takes 4, and 4 of A's steps equal 6 of B's; how many steps will each take before they are together?

We find that while B takes 6 steps A will take 3 steps, and since 6 of B's = 4 of A's, A will lose 1 step every time he takes 3 steps and B takes 6; hence to lose 10 steps, the distance he was ahead, A will take 10 times 3, or 30 steps, and B will take 10 times 6, or 60 steps.

4. Said E to F, my age is 5 years more than yours, but 4 years ago my age was $\frac{1}{2}$ of what yours will be 4 years hence; what was the age of each?

E's age, minus 4 years, is $\frac{1}{2}$ of F's, + 2 years; but E's age = F's + 5 years, and E's age, minus 4 years = -F's age 1 + 1 year; hence F's age + 1 year = $\frac{1}{2}$ of F's, + 2 years; hence $\frac{1}{2}$ of F's age is 1 year, F's age is 2 years, and E's 7 years.

Sol. 2d.—As by the first condition E was 5 years older than P, and by the second 8 years more are added to F's age than to E's, therefore, 4 years hence, F will be 8 - 5, or 3 years older than E was 4 years ago; but since F's then was twice E's 4 years ago, the difference was equal to E's age 4 years ago, which was 3 years; hence E is now 7 years, and F 2 years old.

5. A woman bought 10 meters of silk at the rate of $\$4$ a meter, but finding some of it damaged, for it she only paid $\$1$ a meter, and thus paid $\$28$; how many meters were damaged?

The loss was $\$12$ on the whole, and on each yard $\$4$ - $\$3$, or $\$1$; hence there were 4 damaged yards.

6. C and D ran from the same point in the same direction, and when D had run 40 meters, $\frac{1}{9}$ of the distance C had run equaled the distance he was ahead of D; how much did C, in running 40 meters, gain on D?

C gains $\frac{1}{9}$ of the distance he runs; hence in running 40 rods, he gains $\frac{1}{9}$ of 40 rods, or $4\frac{4}{9}$ rods,

7. A boy bought some peaches at 4 cents each and 3 times as many pears at 2 cents each, and sold them all at 6 cents each, and thus gained 28 cents; how many of each did he buy?

We find he gained on 1 peach and 3 pears 14 cents; hence, to gain 28 cents, he must sell 2 peaches and 6 pears.

8. My friend's watch loses 2 minutes in 3 hours, and mine gains 20 minutes a day; they were set by

correct time yesterday noon, and are now half an hour apart; what time is it?

My friend's watch loses $\frac{2}{3}$ of a minute in an hour while mine gains $\frac{5}{6}$ of a minute in an hour; hence in 1 hour after they were set they will be $\frac{2}{3} + \frac{5}{6}$, or $\frac{3}{2}$ of minute apart, and to be 30 minutes apart will require $30 \div \frac{3}{2}$, or 20 hours; hence the time is 20 hours from yesterday noon, or 8 A.M. to-day.

9. A, B, and C can mow a field in 20 days, A and B in 30 days, and B and C in 40 days; after the three had worked 5 days, A and C finished it; in what time was it completed?

In 5 days they mowed $\frac{1}{4}$ of the field; C can do $\frac{1}{20} - \frac{1}{30}$, or $\frac{1}{60}$ in 1 day, and A can do $\frac{1}{20} - \frac{1}{40}$, or $\frac{1}{40}$ in 1 day; hence A and C can do $\frac{1}{60} + \frac{1}{40}$, or $\frac{1}{24}$ in 1 day, and they can do $\frac{3}{4}$ in $\frac{3}{4} \div \frac{1}{24}$, or 18 days.

10. A person, when asked the time of day, replied that $\frac{2}{3}$ of the time past midnight, 2 hours ago, equaled $\frac{1}{2}$ of the time to midnight, 3 hours and 20 minutes hence; what is the time.

By the conditions, $\frac{8}{5}$ of the time to midnight, 3 hours and 20 minutes hence = $18\frac{2}{3}$ hours; hence the time past midnight, 2 hours ago, is 7 hours; the time now, 9 A.M.

11. A man left \$5000 to his wife, son, and daughter, so that if the daughter died before becoming of age the widow should have $\frac{1}{3}$ of the savings, but if the son died she should have $\frac{2}{3}$ of it; what is the shares of the son and daughter if the widow dies.

We find that 5 times the daughter's share = the sum they together have, or \$5000; hence the daughter has \$1000; the son, \$4000.

12. A pole whose length was 44 meters was broken into two unequal parts; if the shorter be increased by 3 meters, and the longer be diminished by 5 meters, the first will be $\frac{1}{2}$ as long as the second; what is the length of each part.

If the length of the shorter is increased by 3 feet, and the length of the longer be diminished by 5 feet, the length of the pole will be 42 feet; $\frac{3}{2}$ of the length of the longer = 42 feet; hence the longer is $28 + 5$, or 33 feet; the shorter $44 - 3$, or 11 feet.

13. A staff whose length is 33 meters is in the air and water; and the length in the air, - 2 meters, equals 4 times the length in the water, + 6 meters; what is the length in the air.

14. Two years ago Mr. Smith was 5 times as old as his son John will be 2 years hence, and 3 years hence his age will equal 15 times John's age 3 years ago; what is the age of each.

Mr. Smith's age minus 2 years = 5 times John's age, + 10 years, or Mr. Smith's age = 5 times John's age + 12 years; and by the second condition, Mr. Smith's age + 3 years = 15 times John's age, - 45 years, or Mr. Smith's age now = 15 times John's age, - 48 years; hence 5 times John's age, + 12 years = 15 times John's age, - 43 years, or 10 times John's age = 60 years, John's age is 6 years, and Mr. Smith's age 42 years.

15. Harry gave $\frac{1}{4}$ of his money, lacking 3 cents, to James, $\frac{1}{3}$ of the remainder, lacking 2 cents, to Willie, and $\frac{1}{2}$ of the remainder, lacking 1 cent, to Charles, and then had 8 cents remaining; what was Harry's money before his gifts?

By the conditions of the problem, $\frac{1}{4}$ of his money + 3 cents = 8 cents; hence $\frac{1}{4}$ of his money = 5 cents, and his money = 20 cents.

16. Jordan gave $\frac{1}{5}$ of his money, plus 4 cents, to John, $\frac{1}{4}$ of the remainder, plus 3 cents, to George, and $\frac{1}{3}$ of what now remained, plus 2 cents, to Jackson, and found he had $\frac{1}{5}$ as much as at first; how much money did he have at first?

By the condition, $\frac{2}{5}$ of his money minus 6 cents = $\frac{1}{5}$ of his money; hence he had 30 cents at first.

17. A woman, had two watches, bought a chain for \$20. If the chain is put on the silver watch, their value will be $\frac{1}{3}$ as much as the gold watch; but if it be put on the gold watch, they will be worth 7 times as much as the silver watch; what was the value of each watch?

By a condition of the problem, 7 times the value of the silver watch equals the value of the gold watch + \$20, but the gold watch equals 3 times the value of the silver watch, + \$60, therefore 7 times the value of the silver watch equals 3 times the value of the silver watch. + \$80; hence 7 times - 3 times, or 4 times the value of the silver watch equals \$80; therefore once the value of the silver watch equals \$20, and the gold watch is worth

18. Two boats leave a wharf at the same time for the same point; it takes one boat 10 hours to reach it, and the other boat sails 5 times as fast going, and 10 times as fast returning; when do they meet?

The second boat would reach her destination in 2 hours, and the first boat would have 10 - 2, or 8 hours' journey remaining; in returning, the second boat would go over 10 times the distance the first goes, and together they would go 11 times the distance the first goes, or the distance passed by the first in 8 hours; the first goes $\frac{1}{11}$ of the distance, which requires $\frac{8}{11}$ of an hour; hence they meet in $\frac{28}{11}$ hours from the time of starting.

19. Paid $\$34\frac{1}{2}$ for corn at $\$ \frac{3}{4}$, wheat at \$1, and oats at $\$ \frac{1}{2}$ a kg; sold $\frac{2}{3}$ of the corn and $\frac{1}{2}$ of the wheat for 50 percent advance, gaining on the corn $\frac{2}{3}$ as much, as on the wheat, and on the sale the cost of the oats; how much of each did I buy?

By selling $\frac{2}{3}$ of the corn and $\frac{1}{2}$ of the wheat at 50 percent advance, I gained $\frac{1}{3}$ of the cost of the corn and $\frac{1}{4}$ of the cost of the wheat, and since I gained on the corn $\frac{2}{3}$ as much as on the wheat, $\frac{1}{3}$ of the cost of the corn is $\frac{2}{3}$ of $\frac{1}{4}$, or $\frac{1}{6}$ of the cost of the wheat; and the cost of the corn equals $\frac{1}{2}$ of the cost of the wheat; the whole gain was $\frac{1}{6} + \frac{1}{4}$, or $\frac{5}{12}$ of the cost of the wheat, which is the cost of the oats; hence $\frac{6}{12} + \frac{5}{12} + 1\frac{2}{12}$, or $\frac{23}{12}$ of the cost of the wheat equals $\$34\frac{1}{2}$; the wheat cost \$18, the corn \$9, and the oats $\$7\frac{1}{2}$; therefore I bought 18 bushels of wheat, 12 of corn, and 15 of oats.

20. A person has two cups and a cover which weighs 30 ounces. If the first cup be covered it will weigh twice as much as the second, but if the second cup be covered it will weigh 3 times as much as the first; what is the weight of each cup?

The first cup weighs twice the second, - 30 oz., and the second cup weighs 3 times the first, - 30 oz., but 3 times the first equals 6 times the second, - 90 oz..

21. A and B were engaged by a Pennsylvania farmer to dig 100 meters of ditch for \$100; and since the part which A was to dig was more difficult of excavation than that which B dug, it was agreed that A should receive 10 shillings per rod, and B 6 shillings per rod. They each received \$50 for their labor. How many meters did each dig?

6.12 Lesson 12 - Miscellaneous Problems

A PERSON, when asked the hour of the day, replied that 2 hours ago the time past noon was $\frac{1}{3}$ of the time to midnight 2 hours hence; what is the time?

Solution—We find $\frac{4}{3}$ of the time to midnight 2 hours hence + 2 hours + 2 hours, equals 12 hours; so, $\frac{4}{3}$ of the time to midnight, 2 hours hence, equals 8 hours, and $\frac{1}{3}$ of the time to midnight 2 hours hence, which was the time past noon 2 hours ago, equals $\frac{1}{4}$ of 8 hours, or 2 hours; hence the time now is 2 hours + 2 hours, or 4 hours past noon, or 4 o'clock P.M.

2. A man went to a store and spent 20 dollars, and then spent $\frac{3}{4}$ as much of the remaining, now has $\frac{1}{3}$ as much as he had at first, minus \$1; how much did he have at first?

3. A is 10 steps before B, and takes 2 steps while B takes 4, and 4 of A's steps equal 6 of B's; how many steps will each take before they are together?

4. Said E to F, my age is 5 years more than yours, but 4 years ago my age was $\frac{1}{2}$ of what yours will be 4 years hence; what was the age of each?

Sol. 2d.—As by the first condition E was 5 years older than P, and by the second 8 years more are added to F's age than to E's, therefore, 4 years hence, F will be 8 - 5, or 3 years older than E was 4 years ago; but since F's then was twice E's 4 years ago, the difference was equal to E's age 4 years ago, which was 3 years; hence E is now 7 years, and F 2 years old.

5. A woman bought 10 meters of silk at the rate of \$4 a meter, but finding some of it damaged, she only paid \$1 a meter, and thus paid \$28 for all; how many meters were damaged?

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8. My friend's watch loses 2 minutes in 3 hours, and mine gains 20 minutes a day; they were set by correct time yesterday noon, and are now half an hour apart; what time is it?