

Percents and Decimals with Verbal Math Lesson
Step-by-step without pencil or paper

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Introduction

This book teaches your child the very important skill of conceptually understanding and working with percents. We first start with decimal manipulation skills and then lead these into working with percents. All to be done verbally with teacher or parent doing the reading of the problem and child responding with the answer.

We strongly suggest that you go through the Fraction book first before tackling this book.

If student has difficulty, please go over the solution as given to help student understand the steps. It is essential that you start this book with lesson one, even if it seems too easy.

This is our first edition and it may have typos and errors. Please be so kind as to let us know as you find these.

With best wishes,

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Lesson 1 - Adding and Subtracting Decimals

The whole number system has series of integers, 1, 2, 3 In whole numbers, we do not have numbers that are less than one. Fractions that are made up of integers is a way of writing numbers that are smaller than one or fall in between the integers.

Decimals is an even nicer way of writing fractions and whole numbers. With decimals we can write whole numbers, and both rational and irrational numbers. It is a single system for writing all numbers large or small. It is the most used and preferred system of writing numbers.

The word decimal has the root "deci" which means 10. The position of the decimal tells us the value of the number as a multiple of 10.

Take 2.45 and 24.5. By moving the decimal to the right (from between 2 and 4 to between 4 and 5), the number became 10 times bigger.

In 12.8 and 1.28, moving the decimal to the left by one place, made the number 10 times smaller.

In 1245.6 and 12.456, moving the decimal to the left by two spaces, made the number 100 times smaller.

Each movement of the decimal to the right makes the number 10 times bigger.

Each movement of the decimal to the left makes the number 10 times smaller.

1. Where would you put the decimal to make these numbers 10 times bigger?

10.3; 15.7; 345.8; 85.7; 33.89; 9.22.

Ans: One place to the right of where it is right now. 103.; 157.; 3458.; 857. ;338.9; 92.2.

2. Where would you put the decimal to make these numbers 100 times bigger?

10.311; 15.722; 345.83; 85.700; 33.890; 922.6.

Ans: Two places to the right of where it is right now. 103.11; 157.22; 3458.3; 857.00; 3389.0; 9226.

3. Where would you put the decimal to make these numbers 1000 times bigger?

10.311; 15.722; 345.83; 85.700; 33.890; 922.6.

Ans: Three places to the right of where it is right now. 1031.1; 1572.2; 34583.; 85700.; 3389.0; 92260.

4. Where would you put the decimal to make these numbers 10 times smaller?

10.3; 15.7; 345.8; 85.7; 33.89; 9.22.

Ans: One place to the left of where it is right now. 1.03; 1.57; 34.58; 8.57; 3.389; .922.

5. Where would you put the decimal to make these numbers 100 times smaller?

100.0; 115.0; 345.0; 805.7; 303.0; 92.

Ans: Two places to the left of where it is right now. 1.00; 1.15; 3.45; 8.057; 3.03; 9.2.

6. Where would you put the decimal to make these numbers 1000 times smaller?

1450.0; 1455.0; 32245.0; 8665.0; 3903.0; 9122.0.

Ans: Three places to the left of where it is right now. 1.450; 1.455; 32.245; 8.665; 3.903; 9.122.

Fractions can be written as decimal numbers by converting the fraction such that its denominator is 10. The fraction $\frac{3}{5}$ can be written as $\frac{6}{10}$ and this in turn can be written as 0.6 in the decimal form.

In decimals, $\frac{1}{5}$ is same as 0.2. Similarly we write 0.3 the decimal number in fractions as $\frac{3}{10}$ and decimal number 0.5 as $\frac{5}{10}$ or $\frac{1}{2}$. 0.1 is written as $\frac{1}{10}$.

Decimals can be bigger than 1 or smaller. When a decimal is smaller than 1, it has a 0 or nothing in front of the decimal point. A decimal that has a number larger than 0 on the left side of the decimal is larger or equal to 1.0.

7. Which of these decimals are less than 1.0? 1.01; 1.00; 0.101; 0.001, 0.109, 0.456, 4.56? **Ans:** no; no; yes; yes; yes; no.

The first digit after the decimals point is called the tenths.

8. Which is the tenths digit in these numbers? 0.11; 2.33; 4.55; 2.2; .002; 0.003; 9.2? **Ans:** 1; 3; 5; 2; 0; 0; 2.

Hundredths have two digits after the decimal point.

Thousandths have three digits after the decimal point.

There can be any number of zeros after the decimal point.

We add and subtract decimal numbers the same way as we add and subtract regular (called integers) number.

9. What is 0.1 plus 0.1? **Ans:** 0.2

10. Add these decimal numbers.

$0.1 + 0.1$ **Ans:** 0.2 $0.7 + 0.7$ **Ans:** 1.4
 $0.1 + 0.2$ **Ans:** 0.3 $0.8 + 0.8$ **Ans:** 1.6
 $0.2 + 0.3$ **Ans:** 0.5 $0.9 + 0.9$ **Ans:** 1.8
 $0.3 + 0.4$ **Ans:** 0.7 $0.01 + 0.1$ **Ans:** 0.11
 $0.4 + 0.5$ **Ans:** 0.9 $0.02 + 0.1$ **Ans:** 0.12
 $0.5 + 0.5$ **Ans:** 1.0 $0.03 + 0.01$ **Ans:** 0.04
 $0.5 + 0.6$ **Ans:** 1.1 $0.05 + 0.05$ **Ans:** 0.10
 $0.6 + 0.6$ **Ans:** 1.2 $0.01 + 0.09$ **Ans:** 0.10

11. Add these decimal numbers.

$0.1 + 0.1$ **Ans:** 0.2 $0.7 + 0.7$ **Ans:** 1.4
 $0.1 + 0.2$ **Ans:** 0.3 $0.8 + 0.8$ **Ans:** 1.6
 $0.2 + 0.3$ **Ans:** 0.5 $0.9 + 0.9$ **Ans:** 1.8
 $0.3 + 0.4$ **Ans:** 0.7 $0.01 + 0.1$ **Ans:** 0.11
 $0.4 + 0.5$ **Ans:** 0.9 $0.02 + 0.1$ **Ans:** 0.12
 $0.5 + 0.5$ **Ans:** 1.0 $0.03 + 0.01$ **Ans:** 0.04
 $0.5 + 0.6$ **Ans:** 1.1 $0.05 + 0.05$ **Ans:** 0.10
 $0.6 + 0.6$ **Ans:** 1.2 $0.01 + 0.09$ **Ans:** 0.10

12. Add these decimal numbers.

$1.1 + 0.1$ **Ans:** 1.2 $2.7 + 2.7$ **Ans:** 5.4
 $1.1 + 0.2$ **Ans:** 1.3 $3.8 + 3.8$ **Ans:** 7.6
 $2.2 + 0.03$ **Ans:** 2.23 $1.9 + 1.9$ **Ans:** 3.8
 $10.3 + 0.4$ **Ans:** 10.7 $10.01 + 0.1$ **Ans:** 10.11
 $9.4 + 0.5$ **Ans:** 9.9 $10.02 + 0.1$ **Ans:** 10.12
 $1.5 + 0.5$ **Ans:** 2.0 $1.03 + 10.01$ **Ans:** 11.04
 $2.5 + 1.6$ **Ans:** 4.1 $10.05 + 0.05$ **Ans:** 10.10
 $1.6 + 1.6$ **Ans:** 3.2 $10.01 + 10.09$ **Ans:** 20.10

13. Subtract these decimal numbers.

$1.1 - 0.1$ **Ans:** 1.0 $2.7 - 0.8$ **Ans:** 1.9
 $1.1 - 0.2$ **Ans:** 0.9 $1.8 - 0.02$ **Ans:** 1.78
 $0.2 - 0.03$ **Ans:** 0.17 $1.9 - 0.001$ **Ans:** 1.899
 $1.3 - 0.4$ **Ans:** 0.9 $100 - 0.001$ **Ans:** 99.998
 $1.4 - 0.5$ **Ans:** 0.9 $10.02 - 0.1$ **Ans:** 9.98
 $2.5 - 0.5$ **Ans:** 2.0 $1.03 - 0.01$ **Ans:** 1.02
 $2.5 - 1.6$ **Ans:** 0.9 $10.05 - 0.05$ **Ans:** 10.00
 $1.6 - 1.5$ **Ans:** 0.1 $10.01 - 1.05$ **Ans:** 8.95

8. Subtract these tenths decimal numbers.

$1.0 - 0.1$	Ans: 0.9	$10 - 2.7$	Ans: 7.3
$1.0 - 0.01$	Ans: 0.97	$10 - 3.8$	Ans: 6.2
$2.0 - 0.05$	Ans: 1.95	$10 - 1.9$	Ans: 8.1
$2.0 - 0.005$	Ans: 1.995	$10.11 - 0.1$	Ans: 10.01
$3.0 - 0.008$	Ans: 2.992	$10.02 - 0.1$	Ans: 9.92
$3.0 - 0.08$	Ans: 2.92	$10.01 - 9.01$	Ans: 1.0
$4.0 - 0.01$	Ans: 3.91	$10.05 - 0.05$	Ans: 10.00
$10.001 - 10.00$	Ans: 0.001	$10.05 - 10.04$	Ans: 10.01

Lesson 2 - Multiplying Decimals

WHAT is 0.5 times 0.2?

Solution—First convert these decimal numbers to fractions. 0.5 is equal to $\frac{1}{2}$ and 0.2 is equal to $\frac{1}{5}$. Multiply the two fractions and we get $\frac{1}{10}$, which can be written in decimals as 0.1.

To multiply decimals directly without converting them to fractions, we just multiply the numbers as we would do with 5 and 2 except we have to know where to put the decimal.

In this case we get $5 \times 2 = 10$. Now count the decimal places for the two digits we multiplied. Both are one place to the right of the decimal. So for the product, we would put decimal at two places from the right or the end of the number. For 10 which is product of 5 and 2, we put the decimals two place to the right of the end of the number 10 and we get 0.10. We can also write 0.10 as 0.1, because unlike the 0 at the end of an integer, the 0 at the right end of a decimal number can be dropped and is called *not significant*.

Example: What is 2.5 times 2?

Solution—First forget about all the decimals that are there. Pretend you only have two whole numbers. The product of 25 and 2 is 50. Now to figure out where to put the decimal, count the numbers to the right of the decimals. There is only one number past the decimal for 2.5 and none for 2. So we put the decimal one place from the right of the end of the number or 5.0. Again we can drop the 0 to the right of the decimal and we can write 5.0 as 5. or just plain 5.

Example: What is 0.1 times 3.463?

Solution—The product of 1 and 3463 is 3463. Let's count the numbers to the right of the decimals, there is only one for 0.1 and three for 3.463. So we put the decimal one plus three, four places to the right from the end of the number or .3463. We can also write this as 0.3463 which is exactly the same thing.

Example: What is 0.001 times 4.55?

Solution—The product of 1 and 455 is 455. Let's count the numbers to the right of the decimals, there are three for 0.001 and two for 4.55. So we put the decimal three plus two, or five places to the right from the end of the number 455. But there are only three digits in the number, so we just add two zeros in front of 4 and get the answer of 0.00455.

1. Multiply these decimal numbers.

$0.1 \times 1 = 0.1$	$0.1 \times 0.1 = 0.22$	$0.1 \times 1.9 = 0.001$
$0.1 \times 2 = 0.2$	$0.1 \times 0.02 = 0.002$	$0.1 \times 30 = 0.22$
$0.1 \times 20 = 2.0$	$0.1 \times 0.002 = 0.0002$	$0.1 \times 40 = 0.22$
$0.1 \times 400 = 40.0$	$0.1 \times 0.0032 = 0.00032$	$0.1 \times 400 = 0.22$
$0.1 \times 1200.0 = 120.0$	$0.1 \times 5.5 = 0.55$	$0.1 \times 6000 = 0.22$

2. Multiply these decimal numbers.

$2 \times 1 = 2$	$0.1 \times 0.1 = 0.22$	$2.5 \times 2.0 = 50.00$
$3 \times 0.2 = 0.6$	$0.1 \times 0.2 = 0.02$	$2.5 \times 30 = 75.0$
$0.1 \times 4 = 0.4$	$0.1 \times 0.02 = 0.002$	$2.5 \times 40 = 100.0$
$0.1 \times 40 = 4.0$	$0.1 \times 0.032 = 0.0032$	$5.0 \times 400 = 2000.$
$0.1 \times 12000.0 = 1200.0$	$0.1 \times 2.2 = 0.22$	$10.0 \times 0.6 = 6.0$

3. Multiply these decimal numbers.

$0.01 \times 1 = 0.1$	$0.01 \times 4 = 0.04$	$0.01 \times 0.7 = 0.007$
$0.01 \times 2 = 0.2$	$0.01 \times 40 = 0.40$	$0.01 \times 7 = 0.07$
$0.01 \times 20 = 2.0$	$0.01 \times 300 = 3.00$	$0.01 \times 60 = 0.60$
$0.01 \times 400 = 40.0$	$0.01 \times 5000 = 50.00$	$0.01 \times 400 = 4.00$
$0.01 \times 12.0 = 0.12$	$0.01 \times 10000 = 100.00$	$0.01 \times 6000 = 60.00$

4. Multiply these decimal numbers.

$0.001 \times 1 = 0.001$	$0.01 \times 0.1 = 0.001$	$0.2 \times 1.9 = 0.38$
$0.001 \times 0.2 = 0.0002$	$0.001 \times 0.02 = 0.0002$	$0.4 \times 30 = 1.2$
$0.001 \times 20 = 0.020$	$0.1 \times 0.002 = 0.0002$	$0.03 \times 40 = 1.2$
$0.001 \times 400 = 0.4$	$0.01 \times 0.032 = 0.0032$	$0.004 \times 400 = 1.6$
$0.001 \times 12000.0 = 12.00$	$0.01 \times 5.05 = .0505$	$0.05 \times 6000 = 300$

5. What is 0.25 multiplied by 10? **Ans:** 2.5

6. What is 0.25 multiplied by 100? **Ans:** 25

7. What is 0.125 multiplied by 10? **Ans:** 1.25

8. What is 0.125 multiplied by 100? **Ans:** 12.5

9. What is 0.1 multiplied by 10? **Ans:** 1.0

10. What is 0.1 multiplied by 100? **Ans:** 10.0

11. What is 0.01 multiplied by 10? **Ans:** 0.1

12. What is 0.01 multiplied by 100? **Ans:** 1.

13. What is 0.125 divided by 10? **Ans:** 0.0125

14. What is .0125 multiplied by 100? **Ans:** 1.25

18. Multiply 0.1×1.05 ? **Ans:** 0.105

- 19.** What is 10×0.1 . **Ans:** 1.0
- 20.** What is 100×0.1 . **Ans:** 10.0
- 21.** What is 1000×0.1 . **Ans:** 100
- 22.** What is 0.2×20 . **Ans:** 4.0
- 23.** What is 0.02×20 . **Ans:** 0.4
- 24.** What is 0.035×2.0 **Ans:** 0.70.
- 25.** What is 234×0.01 . **Ans:** 0.234
- 26.** What is 45×0.001 . **Ans:** 0.045
- 27.** What is 68.7×0.01 . **Ans:** 0.687

Lesson 3 - Dividing Decimals

DIVIDE 10 by 0.1.

Solution—First let's see what this means by using fractions. 0.1 is same as $\frac{1}{10}$. Dividing 10 by $\frac{1}{10}$ is same as $10 \div \frac{1}{10} = 10 \times 10 = 100$.

You should remember that when we divide a number by a decimal number that is smaller than one, the answer is larger than the numerator. Just as the answer 100 in this problem is bigger than 10.

2. If you divide $10 \div 0.2$, is the answer bigger than 10 or smaller? What is the answer?

Solution—We are dividing by a decimal number less than 1, so the answer will be bigger than 10.

0.2 is same as $\frac{2}{10}$. Dividing 10 by $\frac{2}{10}$ is $10 \times \frac{10}{2} = 50$.

Fractions that have a 5 or 10 in the denominator are easy to convert to decimals. But what about a fraction like $\frac{3}{4}$? To convert this, we need to do division.

3 is less than 4, which means, 4 does not go into 3, and we write down this as $0.$ our starting point. The remainder now is 3. Here we borrow a 10 which is multiplied by 3, the remainder to become 30. Now divide 30 by 4. 4 goes into 30 7 times. So now we have 0.7 . The remainder is 2. Again we borrow a 10 and multiply it by the remainder 2. 4 goes into 20 5 times with no remainder and the division is complete with the answer 0.75 as the decimal number which is same as the fraction $\frac{3}{4}$.

With certain fractions such as 1 divided by 3, we are never be able to end the division. In this case we end after some number of times after the decimal. $\frac{1}{3}$ is equal to 0.3333333... but often as in this book, we will stop at 2 or 3 places.

What is fraction $\frac{2}{3}$ in decimal? Ans: 0.666667....

The reason we put a 7 at the end is that we know that $\frac{1}{3} + \frac{2}{3} = 1.0$, and if we say that $\frac{1}{3}$ is 0.3333 then $\frac{2}{3}$ must be 0.6667 so the sum adds to 1.0.

For all division problems, before asking for the answer, ask if the answer will be bigger or smaller than the numerator.

- 3.** Divide $1 \div 0.2$. **Ans:** 5.
- 4.** Divide $1 \div 0.01$. **Ans:** 100
- 5.** Divide $1 \div 0.002$. **Ans:** 500
- 6.** Divide $1 \div 0.1$. **Ans:** 10
- 7.** Divide these.

Lesson 6 - Proportions and Fractions

WHAT number is the same proportion of 100 as 3 is to 6?

Ans: 3 is one half of 6. One half of 100 is 50.

2. What number is the same proportion of 100 as 1 is to 10?

Ans: 1 is $\frac{1}{10}$ of 10. $\frac{1}{10}$ of 100 is 10.

3. What number is the same proportion of 100 as 1 is to 5? **Ans:** 20.

4. What number is the same proportion of 100 as 4 is to 5? **Ans:** 80.

Ans: 4 is $\frac{4}{5}$ of 5. $\frac{4}{5}$ of 100 is 80.

5. What number is the same proportion of 100 as 3 does to 5?

Ans: 3 is $\frac{3}{5}$ of 5. $\frac{3}{5}$ of 100 is 60.

6. What number is the same proportion of 100 as $2\frac{1}{2}$ is to 5?

Ans: $2\frac{1}{2}$ is to 5 as $\frac{1}{2}$. $\frac{1}{2}$ of 100 is 50.

7. What number is the same proportion of 100 as $3\frac{1}{2}$ does to 7? **Ans:** 50.

8. What number is the same proportion of 100 as 1 is to $2\frac{1}{2}$? **Ans:** 40.

Ans: 1 is to $2\frac{1}{2}$ as $\frac{2}{5}$. $\frac{2}{5}$ of 100 is 40.

9. What number is the same proportion of 100 as $\frac{1}{2}$ is to $2\frac{1}{2}$?

Ans: $\frac{1}{2}$ is to $2\frac{1}{2}$ as $\frac{2}{10}$. $\frac{2}{10}$ of 100 is 20.

10. What number is the same proportion of 100 as $4\frac{1}{2}$ is to 10?

Ans: $4\frac{1}{2}$ is to 10 as $\frac{9}{20}$. $\frac{9}{20}$ of 100 is 45.

11. What number is the same proportion of 100 as 3 is to 2? **Ans:** 150.

12. What number is the same proportion of 100 as $\frac{1}{2}$ is to 5?

Ans: $\frac{1}{2}$ is to 5 as $\frac{1}{10}$. $\frac{1}{10}$ of 100 is 10.

13. What number is the same proportion of 100 as $2\frac{1}{2}$ is to $\frac{1}{4}$?

Ans: $2\frac{1}{2}$ is to $\frac{1}{4}$ as 10. 10 times of 100 is 1000.

14. What number is the same proportion of 90 as $2\frac{1}{3}$ is to 21?

Ans: $2\frac{1}{3}$ is $\frac{7}{3}$. $\frac{7}{3}$ is to 21 as $\frac{1}{9}$. $\frac{1}{9}$ 90 is 10.

15. What number is the same proportion of 100 as $2\frac{1}{5}$ does to 22?

Ans: $2\frac{1}{5}$ is $\frac{11}{5}$. $\frac{11}{5}$ is to 22 as $\frac{1}{10}$. $\frac{1}{10}$ 100 is 10.

16. What number is the same proportion of 100 as $\frac{3}{8}$ does to 15?

Ans: $\frac{3}{8}$ is to 15 as $\frac{1}{40}$. $\frac{1}{40}$ 100 is $2\frac{1}{2}$.

17. What number is the same proportion of 100 as $\frac{5}{8}$ does to 5?

Ans: $\frac{5}{8}$ is to 5 as $\frac{1}{8}$. $\frac{1}{8}$ 100 is $12\frac{1}{2}$.

Lesson 12 - Finding Percent of a Number (33.33%, 66.667%)

The FRACTION $\frac{1}{3}$ is equal to what decimal number? **Ans:** 0.333

2. The decimal number 0.333 is equal to what percent? **Ans:** 33.33%

3. What part of a circle is $33\frac{1}{3}\%$ of it? **Ans:** One third.

4. How many minutes in $33\frac{1}{3}\%$ of 1 hour? **Ans:** One third of 60 minutes, 20 minutes.

5. How many dollars in $33\frac{1}{3}\%$ of \$36? **Ans:** One third of 36 dollars, 12 dollars.

6. If a man gave to a charity $\frac{1}{3}$ of his property, what percent of his property did he give away? **Ans:** 33.33%

7. In an orchard of 42 trees $33\frac{1}{3}\%$ are pear trees. How many pear trees are there in the orchard? **Ans:** One third of 42 or 14 trees.

8. What percent of a circle is $\frac{1}{3}$ of the circle? **Ans:** 33.33%

9. The fraction $\frac{2}{3}$ is equal to what decimal number? **Ans:** 0.667

10. The decimal number 0.667 is equal to what fraction? **Ans:** $\frac{2}{3}$.

11. The sum of $\frac{2}{3}$ and $\frac{1}{3}$ is equal to what number? **Ans:** 1.

12. The sum of $33\frac{1}{3}\%$ and $66\frac{2}{3}\%$ is what percent? **Ans:** 100%

13. If I took an apple and cut it in three equal pieces, what percent is one of the pieces to the whole? **Ans:** $33\frac{1}{3}\%$

14. What percent of a circle is $\frac{2}{3}$ of the circle? **Ans:** Double of $33\frac{1}{3}\%$ or $66\frac{2}{3}\%$.

15. What fraction of a circle is $66\frac{2}{3}\%$? **Ans:** $\frac{2}{3}$.

16. A pizza was cut in 12 pieces. How many pieces are contained in $66\frac{2}{3}\%$? **Ans:** One third of 12 is 4. $66\frac{2}{3}\%$ is equal to two-thirds, there are 8 pieces in $66\frac{2}{3}\%$.

17. $\frac{1}{6}$ is equal to what decimal number? **Ans:** 0.1667

18. $\frac{1}{12}$ is equal to what decimal number? **Ans:** 0.0833

19. Each of the 12 pieces of the pizza is equal to what percent of the whole?

Ans: $\frac{1}{12}$ is equal to 0.0833. Multiply this by 100 and we get 8.33%

20. A girl had 21 pieces of chocolate and gave away $66\frac{2}{3}\%$ of them. How many pieces did she give away? **Ans:** Two thirds or 14 pieces.

21. How many inches in $66\frac{2}{3}\%$ of a yard? **Ans:** A yard has 36 inches. $66\frac{2}{3}\%$ of that is $\frac{2}{3}$ of 36 in. or 24 in.

22. If a boy lost $\frac{1}{3}$ of his homework points for not doing it in pencil,

62. A mother divided her property equally for each of her children. How many children did she have, if each child got 14.3%, 16.667%, 11.1%, 33.33%. 50%, 25%. **Ans:** 7; 6; 9; 3; 2; 4.

63. What is the number of which 12 is 50%? 25%; 10%. **Ans:** 24; 48; 15; 120.

64. What percent of 10 is 1? $1\frac{2}{3}$? $2\frac{1}{2}$? $3\frac{3}{4}$? 4? $6\frac{1}{4}$? 8? **Ans:** 10%, 16.667%, 25, 37.5%, 40%, 62.5%, 80%.

Lesson 13 - Finding Percent of a Number (100%, 1%, 10%)

Its VERY easy to find 100% of a number. It is the number multiplied by 1.0. 10% is also very easy, it is the number divided by 10.

2. 50% of something plus 50% of the same thing equals what? **Ans:** All of something or 100%.

3. What is $\frac{1}{2}$ plus $\frac{1}{2}$? **Ans:** 1.0

4. What $0.5 + 0.5$? **Ans:** 1.0

5. What is $50\% + 50\% = 100\%$

4. What is 25% plus 75%? **Ans:** 100%

5. Add the fractions $\frac{1}{4}$ and $\frac{3}{4}$? **Ans:** 1.0

6. Add the fractions $\frac{1}{2}$ and $\frac{3}{4}$? **Ans:** 1.25

7. My investment increased 100% in price. What does that mean?

Ans: Its price increased by the same amount as the old price. So it is now worth twice as much money as before.

8. What is 100% of 24.5? **Ans:** 24.5

9. 100% of students have had their vaccinations? What does that mean? **Ans:** All of them have had their vaccinations.

10. Can we say that 110% of the students in the class have had their shots? **Ans:** No, the maximum you can have is 100% in this case. No more students are possible.

11. The puppy weighed 6 pounds when she was 4 months old. When she was 6 months old, her weight was 100% more than before. How much does she weigh now?

Ans: 100% of 6 pounds is 6 pounds, so she is now 12 pounds.

12. What does it mean when I say that the hotel is 100% full. **Ans:** All the rooms in the hotel are taken.

13. What does it mean when I say that the hotel is 50% full. **Ans:** Half the rooms in the hotel are taken.

14. 100% is how many times 10%? **Ans:** 10 times.

15. At the end of the year, I received 10% interest on savings of \$2250. How much money do I have now? **Ans:** The interest is \$225. So I have $\$2250 + \$225 = \$2475$.

15. Grandma had 10 pairs of reading glasses and then she bought one more. What is the percent increase in the number of glasses grandma has? **Ans:** 10%

31. What is 1% of 6700? 6300? 10000? 150? 290?

Ans: 67; 63; 100; 1.5; 2.9.

32. What is 1% of 100? 10? 1? 0.1? 0.01?

Ans: 1; 0.1; 0.001; 0.0001; 0.00001.

33. 1% of something is how many times 10% of the same thing? **Ans:**
One tenth

34. 10% of something is how many times 1% of the same thing? **Ans:**
Ten times

35. 100% of something is how many times 10% of the same thing?
Ans: Ten times

36. 100% of something is how many times 1% of the same thing?
Ans: Hundred times

37. 1000% of something is how many times 200% of the same thing?
Ans: Five times

38. What is 1% of one million? **Ans:** 10,000.

39. What is 10% of one million? **Ans:** 100,000.

40. What is 100% of one million? **Ans:** One million.

41. What is 1% of 0.1? **Ans:** 0.001.

42. What is 10% of .01? **Ans:** 0.001.

43. What is 100% .001? **Ans:** 0.001.

44. Which is bigger: 1% of 23.4 or 10% of 2.34? **Ans:** They are
same.

45. A product when packed fits in 40% less space. If it fits in a box
that was 10" tall before packing, what size would the product need when
it is packed? **Ans:** 6"

46. 20% of the cotton is wasted when making fabric. How much
fabric can be made from $2\frac{1}{2}$ ton of cotton? **Ans:** 2 tons of fabric.

47. Out of 600 pounds only 200 pounds is usable, what percent is
usable? **Ans:** 33.33%

48. A batch of 1000 pounds was cleaned for seeds and 450 pounds of
seeds were collected. What percent is that? **Ans:** 45%

49. A girl deposited \$80 last year and \$100 this year in her account.
What is the percent of increase.

Ans: The increase is $100 - 80 = 20$. To find the increase in percent,
divide 20 by the starting value and multiply by 100. $\frac{20}{800} \times 100 = 25\%$

50. Now she has \$180 in her account. How much should she put in
this year to match her percent increase from last year.

Lesson 14 - Finding Percent of a Number (less than 10%)

WHAT is $\frac{3}{4}$ as a decimal number? **Ans:** 0.75

Is $\frac{3}{4}$ equal to 0.75%? **Ans:** No.

2. What is 1% of 4800? **Ans:** It is 4800 divided by 100, or 48.

3. What is 0.5% of 4800? **Ans:** if 1% is 48, then 0.5% is half that or 24.

4. How much is $\frac{3}{4}$ % of 4800? **Ans:** We can do this two ways. One is to multiply 4800 by 0.0075 which seems kind of hard. Second easier method is note that 1% of 4800 is 48. So $\frac{3}{4}$ % is $\frac{3}{4}$ of 48 or 36.

5. What is fraction $\frac{1}{4}$ equal to in a decimal number? **Ans:** 0.25

6. How much is 1% of 1000? **Ans:** 10.

7. How much is $\frac{1}{2}$ % of 1000? **Ans:** 5.

8. How much is $\frac{1}{4}$ % of 1000? **Ans:** 1% of 1000 is 10. Then take $\frac{1}{4}$ of that which is 2.5.

9. How much is $1\frac{1}{4}$ % of 1000? **Ans:** We can do this two ways. One is to multiply 1000 by .0125 or take 1% of 1000 which is 10. Then take $\frac{1}{4}$ of that which is 2.5. Add the two together to get 12.5.

10. How much is $5\frac{1}{4}$ % of 5000? **Ans:** We can do this two ways. One is to multiply 1000 by 0.0525 or take

5% of 5000 which is 250. 1% is 50, so $\frac{1}{4}$ of that is 12.5. Add the two together to get 262.5.

11. How much is $3\frac{1}{4}$ % of 100? 1000? 10000? **Ans:** 3.25; 32.5; 325.

12. How much is $5\frac{1}{4}$ % of 200? 2000? 10000? **Ans:** 10.5; 105; 525.

13. How much is $8\frac{1}{4}$ % of 100? 1000? 10000? **Ans:** 8.25; 82.5; 825.

14. How much is $6\frac{1}{4}$ % of 100? 2000? 10000? **Ans:** 6.25; 125; 625.

15. $\frac{1}{2}$ % of 1000? **Ans:** 5.

16. How much is $3\frac{1}{2}$ % of 100? 1000? 10000? **Ans:** 3.25; 32.5; 325.

17. How much is $8\frac{1}{2}$ % of 100? 1000? 10000? 100000? **Ans:** 3.25; 32.5; 325.

18. How much is $10\frac{1}{2}$ % of 100? 1000? 10000? **Ans:** 3.25; 32.5; 325.

19. The sales tax rate was 3%. The cost of the shoes is \$90. How much is the tax on \$90.

Ans: 10% of 90 is 9. 1% is one tenth of that or 0.9. 3% is 3 times 1%, so the tax is 3 times 0.9 or \$2.7.

20. $\frac{2}{3}$ is equal to what decimal number? **Ans:** 0.667.

Lesson 15 - Finding Percent of a Number

FROM a savings account of \$250, a girl took out 8%. How much money did she take out? **Ans:** \$20.

2. A boy saved \$400 and decided to use $7\frac{1}{2}\%$ of it to buy presents for his family. How much would he spend on the presents?

Ans: $7\frac{1}{2}\%$ is equal to $5\% + 2\frac{1}{2}\%$. 5% of \$400 is \$20. $2\frac{1}{2}\%$ is half that. So he plans to spend \$30.

3. A house purchased for \$50,000 was sold for 40% gain. What was the sale price of the house? **Ans:** \$70,000.

4. 70% of the student were local. Of these 25% were girls and the rest boys. What percent of the class are local boys? **Ans:** 45%

5. A woman invested her money in two mutual funds. In one, she put \$12,000 and received 4% dividend and put \$15,000 in the other and received 3% divided. From which fund did she get more money? **Ans:** First one.

6. A woman lost $37\frac{1}{2}\%$ of her money. She had \$24,000. How much does she have now?

Solution— $37\frac{1}{2}\% = \frac{75}{200} = \frac{3}{8}$. $\frac{3}{8}$ of \$24,000 is \$9,000 the amount she lost.

7. If $62\frac{1}{2}\%$ of \$400 is $\frac{1}{4}$ the cost of the new coat, then what does the coat cost?

Ans: $62\frac{1}{2}\%$ is equal to 50% $12\frac{1}{2}\%$ both of which are easy to figure out. 50% of \$400 is \$200. $12\frac{1}{2}\%$ is one quarter of 50%, so it is one quarter of \$200 or \$50. So \$250 is $\frac{1}{4}$ the cost, so the cost must be \$1000.

8. A family has an income of \$3600 per month and puts 25% of this in savings every month. How much money would they have after 1 years?

Ans: Each month they save 25% of \$3600 which is \$900. In 12 months they have saved \$10,800.

9. A car dealer bought 6 cars for \$18,000 each and sold them for a gain of $33\frac{1}{3}\%$. How much money did he get for each car ? **Ans:** \$3600.

10. Which is more and by how much, 30% of \$400 or 5% of \$3000? **Ans:** 5%. by \$30.

11. I had \$400, I paid 30% for the hotel room, 15% for food and 17.5% for tickets for the game. How much money do I have left? **Ans:** 67.5% is same as fraction $\frac{5}{8}$, so $\frac{5}{8}$ of \$400 is \$250, which leaves \$150..

12. What percent of anything is $\frac{3}{8}$ of it? **Ans:** $\frac{3}{8} \times 100 = 37.5\%$.

13. If someone lost $\frac{3}{8}$ of their marbles, what percent did they lose?

Lesson 16 - Find the Base

- EIGHT is what fraction of 800? **Ans:** $\frac{1}{100}$.
2. 8 is what percent of 800? **Ans:** 1%.
 3. 4 is 1% of what number? **Ans:** 400.
 4. 3 is 1% of what number? **Ans:** 300.
 5. 1.5 is 1% of what number? **Ans:** 150.
 6. 1 is 1% of what number? **Ans:** 100.
 7. 0.1 is 1% of what number? **Ans:** 10.
 8. 0.01 is 1% of what number? **Ans:** 1.0
 9. 2 is what fraction of 100? **Ans:** $\frac{1}{50}$.
 10. 4 is what percent of 800? **Ans:** 2%.
 11. 8 is 2% of what number? **Ans:** 400.
 12. 6 is 2% of what number? **Ans:** 300
 13. 2.5 is 1% of what number? **Ans:** 250.
 14. 2.5 is 2% of what number? **Ans:** 125.
 15. 1 is 2% of what number? **Ans:** 50.
 16. 0.4 is 2% of what number? **Ans:** 20.
 17. 0.02 is 2% of what number? **Ans:** 1.0
 18. 10 is 1% of what number? **Ans:** 100.
 19. 10 is 2% of what number? **Ans:** 50.
 20. 5 is what fraction of 50? **Ans:** $\frac{1}{10}$.
 21. 5 is what fraction of 100? **Ans:** $\frac{1}{20}$.
 22. Fraction $\frac{1}{2}$ is equal to what percent? **Ans:** 50%
 23. Fraction $\frac{1}{20}$ is equal to what percent? **Ans:** 5%
 24. Fraction $\frac{1}{40}$ is equal to what percent? **Ans:** 2.5%
 25. 0.1 is 5% of what number? **Ans:** 5.
 26. 10% of 5 is what number? **Ans:** 0.5.
 27. 10 is 10% of what number? **Ans:** 100.
 28. 10 is 5% of what number? **Ans:** 50.
 29. 5 is 5% of what number? **Ans:** 100.
 30. 5% of 100 is what number? **Ans:** 5.
 31. 2.5% of 100 is what number? **Ans:** 2.5.
 32. 7.5% of 100 is what number? **Ans:** $5 + 2.5 = 7.5$
 33. 7.5% of 20 is what number?

What did it cost and what did he sell it for?

Ans: If $\frac{3}{2}$ is $\frac{2}{5}$ of the cost, then $\frac{1}{5}$ is $\frac{3}{4}$ and the cost is $\$3\frac{3}{4}$ per ft., and the sale price is $\$5\frac{1}{4}$ per ft.

24. If a stock sold for \$21 more than it cost and gave a profit of 30%, what did it cost? **Ans:** \$21 is 30% of \$70, the cost.

25. At what cost must an item costing \$5 be sold to make a profit of 10%? 50%? 100%? 150%? 200%?

Ans: \$5.5; \$7.5; \$10; \$12.5; \$15.

26. A company bought an item for \$5 and sold it for \$6. What percent profit did they make?

Ans: The gain is the difference between \$6 and \$5, which is \$1. The percent gain $\frac{1}{5} 100 = 20\%$.

27. What percent is gained when an item that cost \$50 is sold for \$60? **Ans:** 20%

28. What percent is gained when an item that cost \$500 is sold for \$600? **Ans:** 20%

29. What percent is gained when an item cost \$25 is sold for \$30? **Ans:** 20%

30. We sold our piano for \$3000 which was $\frac{3}{4}$ of what it cost. What was our loss in percent? **Ans:** 25%

31. The car dealer sold a used car for \$7500 which had cost him only \$5000. What's his profit in percent?

Ans: He made \$2500 on cost of \$5000, so his profit is 50%

32. Jimmy sold his candy bar for 75 cents to Jake to who was very hungry. The bar only cost 50 cents. What Jimmy's profit in percent? **Ans:** 50%

33. She sold he bike for $\frac{1}{2}$ of what it cost. What was her loss? **Ans:** 50%

34. A family paid \$3000 for a car. It stopped working soon after and the mechanic said that it would cost more it fix it than it was worth. What was the family's loss? **Ans:** 100%

35. If an item sells \$21 more than its cost and gives a profit of 30%, what was its cost?

Ans: \$21 which is the gain equals 30% of the cost. So the cost is \$70.

36. The business sold the items for $\$2\frac{1}{2}$ more than the cost and made 50% profit. What was the sale price of the item? **Ans:** 50% profit means the item cost \$5, so the sale price is \$7.50.

37. A business by selling the items at \$12 less than what they cost,

Ans: For a principal to treble itself it must gain two-times itself; so at 5% it will require as many years as $\frac{1}{20}$ is contained in $\frac{40}{20}$, or 40; at 10% it requires 20 years; at 20%, 10 y.; at 25%, 8 y.; at 40%, 5 y.; at 50%, 4 y.

18. In what time will a principal quadruple itself, at 5%? At 15% 30? 50? 60? 100?

Ans: For a principal to quadruple itself it must gain 3 times itself; so at 5% it will require as many years as $\frac{1}{20}$ is contained in $\frac{60}{20}$, or 60; at 15% it will require 20 years; at 30% 10 y.; at 50%, 6 y.; at 60%, 5 y.; at 100%, 3 y.

19. The amount of a certain principal, for a certain time, at 5% is \$250, and the amount for the same time, at 8% is \$280. What is the principal and time.

Ans: \$280, the amount at 8% minus \$250, the amount at 5% equals \$30, the interest on the given principal for the given time at 8% minus 5% or 3%.; so the interest at one percent is $\frac{1}{3}$ of \$30, or \$10, and at 5%. 5 times \$10, or \$50, and the principal is \$250 - \$50, or \$200; the time is 5 years.

20. A certain sum of money, on interest, amounts, in a certain time, at 6% to \$310, and at 10% for the same time, to \$350. What is the time and principal.

Ans: \$350 - \$310 = \$40, the interest on the given sum for the given time at 10% minus 6% or 4%.; so the interest at 1% is $\frac{1}{4}$ of \$40, or \$10, and at 6% is \$60; \$310 - \$60 = \$250, the principal; the time is 4 years.

Lesson 38 - Mixed Problems

MY food expenses are 20% of my annual salary and my utilities are 10% of the rest. What is my salary if my food bills are \$6000 more than my utilities?

Ans: My food expenses are 20% of my salary so that leave 80%. The utilities are 10% of the 80% that remains, which is 8% of the total. So \$6000 is the difference between 20% and 8% or 12%. If \$6000 is 12% of a number then to find that number we start with the fraction $\frac{12}{100}$ or $\frac{3}{25}$. If $\frac{3}{25}$ is equal to \$6000, then $\frac{1}{25}$ is \$2000 and $\frac{25}{25}$ is \$50,000.

2. A factory built nuts and bolts in quantities such that 40% of nuts was equal to 75% of the bolts, and of both they made 4600 boxes with equal number in each. How many boxes of each did they make?

Ans: $\frac{40}{100}$ of nuts is equal to $\frac{75}{100}$ of bolts, then $\frac{100}{100}$ of nuts is equal to $\frac{75}{40}$ of bolts. Add both of these and we get $\frac{1150}{400} = \frac{23}{8}$. Now compute fraction of nuts from this total which is $\frac{75}{40} \div \frac{23}{8} = \frac{15}{23}$. Of 4600 boxes, the nuts are $4600 \times \frac{15}{23} = 3000$ boxes and 1600 boxes of bolts.

3. How much money has Adam, if 40% of his money is in coins, 30% of it is in bills, and the rest, which is \$900, he loaned to his sister?

Ans: 30% of his money equals \$900. So the total of his money equals $\frac{900}{30} \times 100 = \3000 .

4. A student sold a computer to his friend at a loss of 20%; the friend sold it to a shop for \$150, and by so doing lost 25%. How much did the student pay for the computer?

Ans: Some times its easy to start at the end. In this case, the friend sold it to the shop for \$150 and lost 25%. Which means \$150 is 75% of the his cost. So the cost to the friend was $\frac{150}{75} \times 100 = \200 . The student sold it for \$200 and lost 20%, so his cost was $\frac{200}{80} \times 100 = \250 .

5. Mr. Brown sold his horse so that 75% of what he paid for it was equal to 60% of the selling price. Did he gain or lose, and what percent?

Ans: Let's say he paid \$100 for the horse. 75% of that is \$75. This is equal to 60% of selling price. So the selling price is equal to $\frac{75}{60} \times 100 = \125 . That means he made money.

6. A dealer wishes to mark the price of a piano so that after giving the buyer a discount of 20%, he can give another 20% off, and still get \$3200 for it. Find the marked price.

Ans: The discount was 20% + 20% of 80% which is equal to 20% + 16% = 36%. The sale price was 64% of list price. The list price was $\frac{3200}{64} \times 100 = \5000 .

Check—The first discount of 20% on \$5000 is \$1000, then the second discount is 20% of \$4000, or \$1800. Total discount is \$1800 and the sale price would be $\$5000 - \$1800 = \$3200$.

7. How must a coat that cost \$300 be marked so that there may be a profit of 20% after a discount of 10% from the marked price?

Ans: The discount is 10%, so the sale price of 90% will be equal to $300 + (20\% \text{ of } 200) = \360 . The sale price should be $\frac{360}{90} \times 100 = \400 .

Check—The sale price is \$400. After 10% off which is \$40, the price is \$360. This is 20% more than the cost of \$300.

8. If a tea dealer always gets for 3 pounds of tea as much as 4 pounds cost, what percent does he gain?

Ans: The price is \$4 for 3 pounds, and the cost is \$3 for the same 3 pounds. So the gain is \$1 out of \$3 or 33.3%

9. How many watermelons at \$5 each must I buy, so that if I sell them at a gain of 20% I may make a profit of \$25?

Ans: The sale price is $5 + 20\% \text{ of } 5 = \6 . To make \$25, I would have to sell 25 watermelons.

10. By selling a box of goods for \$80 less than cost, there was a loss of 10%. For how much should it have been sold to gain 25%?

Ans: \$80 is 10% of the sale price. The sale price is \$800. The cost then must have been \$880. The gain of 25% implies a sale price of $880 + 220 = \$1100$.

11. A mixture of equal parts of two kinds of salt, costing 18 and 22 cents per pound, was sold so as to gain 25%. What was the selling price per pound?

Ans: The average cost of the salt is 20 cents per pound. To sell it at a 25% gain, it is sold at $20 + 25\% \text{ of } 20$ or 25 cents per pound.

12. A field has bikes, scooters, and skateboards, in all 1840. How many of each kind are there if the number of bikes is 60% the number of scooters, and the number of skateboards is 40% of the number of bikes?

Ans: Let's say there are 100 bikes. Number of scooters is $\frac{100}{60} \times 100 = \frac{500}{3}$. The number of skateboards is 40. Of the total fraction of skateboards are: $40 \div (100 + \frac{500}{3} + 40) = \frac{3}{23}$. The fraction of bikes is $100 \div (100 + \frac{500}{3} + 40) = \frac{15}{46}$. The number of bikes is $1840 \times \frac{15}{46} = 600$, the number of skateboards is $1840 \times \frac{3}{23} = 240$ and the rest are scooters.

Check—There are 600 bikes, This is 60% of 1000. So the number of scooters is 1000. The number of skateboards is 40% of 600 or 240. So the total number is $600 + 1000 + 240 = 1840$.