

The
VERBAL
MATH LESSON
FRACTIONS

*Learning Fractions
Step-by-Step
without Pencil or Paper*

Michael Levin M.D.
Charan Langton M.S.

MOUNTCASTLE COMPANY

The
VERBAL
MATH LESSON
FRACTIONS

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Charan Langton M.S.

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Just a little everyday;
 That's the way
Children learn to read and write,
Bit by bit and mite by mite,
 Never anyone, I say,
Leaps to knowledge and its power
Slowly, slowly - hour by hour-
 That's the way;
 Just a little everyday.
 - Ella Wheeler Wilcox

Fractions with Verbal Math Lesson

Michael Levin and Charan Langton

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Introduction

This book was inspired by an old book we found. Printed in 1898, the book called "Mental Math" by Brooks. We tried it on our kids and found it magical. We then searched and realized that so called mental math was a popular method for teaching children math. It bypassed the tediousness of writing and teaches children efficient methods for solving every day problems involving fractions.

In this math course, we bring this time-honored method to help your child become proficient with fractions. Not just be able to do simple problems but to really understand what fractions mean and be able to do complex looking problems quickly and accurately in his head. Follow the lessons and see child do problems that you thought you needed algebra to solve. But no algebra is needed to solve these problems in the way we teach them.

This course introduces a special way of analyzing math problems. Multiplication and division are harder than adding, so we teach a special way to solve problems with mostly addition and subtraction. The method is based on the understanding of unit values. The first couple of problems in each lesson show the process. Have your child follow this method even though he or she may know of other ways. Using this method will make more complicated problems easier to solve. Here is an example of the type of problem the child will be able to do quickly without any writing by the end of this course;

The distance from Mount Joy to Harrisburg is 25 miles, and $\frac{4}{5}$ of this distance is $\frac{5}{8}$ of $\frac{4}{9}$ of the distance from Harrisburg to Minton. What is the distance to Minton?

The main purpose of this course is to help child develop speed and accuracy in solving fraction problems. The problems should be read to the child. The answers are there for you. For classroom use or as home homework, the teacher needs to instruct the parent that the parent should read the problem to the child and must not let them write out the solutions. They may write down the numbers but all calculations should be done in the head without using any writing.

The Verbal Math Lesson contains the type of problems that we all need to solve. These exercises provide training in purely mental computations so that the child can develop a reasonable degree of accuracy and speed.

SOME GUIDELINES

1. You must start this course at Lesson one, even if the problem seem too simple to you.
4. All of these problems can easily be done verbally.
3. Follow the method that is given at the beginning of each lesson. (eBook without answers is available at our website.)
5. Most children can easily do one lesson a day. Some longer lessons can be spread over 2 days.
6. Subordinate speed to accuracy, but do not neglect speed. Ability to answer these problems quickly means mastery.
7. The course is suitable for children starting in 4th grade, depending on the child's ability with arithmetic.

Please let us know if you find any errors in this book. We are also happy to hear of your comments on how we may improve it.

Best wishes,

Michael Levin

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Lesson 2 - *Fifths, Sixths, and Sevenths*

IF you divide an orange into 5 equal parts, what are 1, 2, 3, and 4 of these parts called?

Solution—one fifth, 2 fifths, 3 fifths. 4 fifths.

2. How many fifths are in one orange? *Ans:* 5.

3. What is one fifth of 10? of 25? of 15? of 30? *Ans:* 2; 5; 3; 6.

Solution—one fifth of 10 is 2.

One fifth of 25 is 5.

One fifth of 15 is 3.

One fifth of 30 is 6.

4. What is 2 fifths of 15? of 30? of 45? of 20? *Ans:* 6; 12; 18; 8.

Solution—one fifth of 15 is 3, so 2 fifths is twice that or 6.

One fifth of 30 is 6, so 2 fifths is twice that or 12.

5. What is 3 fifths of 10? of 30? of 25? of 55? *Ans:* 6; 18; 15; 33.

Solution—one fifth of 10 is 2, so 3 fifths is three times that or 6.

6. What is 4 fifths of 55? of 35? of 40? of 50? *Ans:* 44; 28; 32; 40.

7. Mary has 15 oranges, and Rachel has 2 fifths as many. How many oranges does Rachel have?

Solution—one fifth of 15 is 3, so Rachel has 6 oranges.

8. Susan is 25 years old, and her sister is four-fifths as old. How old is her sister?

Ans: 20 years.

9. Rowland is 35 years old, and his wife is four fifths as old. How old is his wife?

Ans: 28 years.

10. A horse cost \$1000, and a saddle cost 3 fifths as much. What is the cost of the saddle?

Solution—One fifth of \$1000 is \$200. 3 fifths is three times as much or \$600. The cost of saddle is \$600.

11. Mary had 40 sheep, she lost 20, and after a lot of searching found 3 fifths of them. How many does she have?

Solution—If Mary lost 20 sheep, she had left $40 - 20$, or 20 sheep. One fifth of 20 is 4, so 3 fifths of 20 which she found is 12. So she has 20 that did not run away and 12 she found or $20 + 12 = 32$, the number of sheep Mary has left.

12. A man had 50 cows, sold 4 fifths of them, and then bought 32 cows. How many does he have? *Ans:* 42 cows.

13. If you divide a melon into 6 equal parts, what are 1, 2, 3, 4, and 5 of these parts called? *Ans:* One sixth, 2 sixths, 3 sixths, 4 sixths, 5 sixths.

14. How many sixths are there in a single thing? *Ans:* 6.

15. What is 2 sixths of 24? of 18? of 36? of 60? *Ans:* 8; 6; 12; 20.

16. What is 3 sixths of 12? of 42? of 30? of 66? *Ans:* 6; 21; 15; 33.

17. What is 4 sixths of 6? of 36? of 48? of 54? *Ans:* 4; 24; 32; 36.

18. What is 5 sixths of 18? of 54? of 24? of 72? *Ans:* 15; 45; 20; 60.

19. Rob had 48 pens, he gave 3 sixths to Megan and 2 sixths to Morgan. How many did he give to both? *Ans:* 24 pens to Megan, 16 to Morgan.

20. What will 5 sixths of 36 meters of cloth cost, at the rate of \$2 a meter? *Ans:* One sixth of 36 is 6. 5 sixths is 5 times that 30 meter. 30 meters of cloth at \$2 per meter will cost \$60.

21. Warren had 12 points, and Oliver had 5 sixths as many less 4. How many points did Oliver have?

Solution—Five sixths of 12 points is 10 points. $10 - 4 = 6$ points, the number of points that Oliver has.

22. Dana, had 60 cards, gave 2 sixths of them to Barton, and 3 sixths to Benton. How many she kept? *Ans:* 10 cards.

23. If 1 meter of speaker wire costs 5 sixths of 36 cents, how many meters can you buy for 60 cents? *Ans:* 2 meters.

24. 2 thirds of \$30 is \$10 less than A's money. How much does Anil have? *Ans:* \$30.

25. Frazier had 40 pens, gave Brown 10, and Seal 2 sixths of the remaining. How many pens does he have left?

Ans: 20 pens.

26. If a melon is divided into 7 equal parts, what are 1, 2, 3, 4, 5, and 6 of these parts called? *Ans:* One seventh, 2 sevenths, 3 sevenths, 4 sevenths, 5 sevenths, 6 sevenths.

27. How many sevenths are there in one? *Ans:* 7.

28. If you give away one seventh of something, how much do you have left? *Ans:* 6 sevenths.

29. If you give away five seventh of something, how much do you have left? *Ans:* 2 sevenths.

30. What is one seventh of 21? of 28? of 42? of 56? *Ans:* 3; 4; 6; 8.

31. What is 2 sevenths of 28? of 49? of 63? of 70? *Ans:* 8; 14; 18; 20.

32. What is 3 sevenths of 14? of 35? of 49? of 28? *Ans:* 6; 15; 21; 12.

33. What is 4 sevenths of 70? of 77? of 63? of 84? *Ans:* 40; 44; 36; 48.

34. What is 5 sevenths of 77? of 91? of 42? of 28? *Ans:* 55; 65; 30; 20.

35. What is 6 sevenths of 35? of 42? of 49? of 140? *Ans:* 30; 36; 42; 120.

36. Jim bought a bicycle for \$70, and sold it for 6 sevenths of its cost. How much money did he lose? *Ans:* \$10.

37. If one half of 10 meters of cable costs \$10, what will one fifth of 10 meters cost?

Solution—One half of 10 meters is 5 meters, and one fifth of 10 meters is 2 meters. If 5 meters cost \$10, 1 meter will cost one fifth of \$10, or \$2, so 2 meters will cost \$4.

38. A computer programmer paid \$70 for a organizer program, and 3 sevenths as much for a manual, and then sold them both for \$90. What was his loss?

Ans: Three sevenths of \$70 is \$30. $\$70 + \30 is \$100, the whole cost. $\$100 - \90 is \$10, the loss.

39. Three sevenths of \$56 is \$6 more than what a book cost. What will 2 books cost at the same rate? *Ans:* \$36

40. Richard had \$240, one third of which he spent for a suit, one fourth for a watch, and one sixth for shoes. How much does he have left?

Ans: One third of 240 is 80, one fourth of 240 is 60, one sixth of 120 is \$40. So he spent $80 + 60 + 40$ and has left $240 - 180 = \$60$.

41. Ashok had \$140. He gave 3 sevenths of it to his cousin, and spent 3 fourths of the rest on

books. How much does he have left?

Ans: He gave to his cousin $\frac{3}{7}$ of 140, or 60. and had left $140 - 60 = 80$. He then spent $\frac{3}{4}$ of 80, or 60, and had left $80 - 60 = \$20$.

Lesson 3 - *Eighths and Ninths*

If anything is divided into 8 equal parts, what is one of these parts called?

Solution—one eighth.

2. What are 2, 3, 4, 5, 6 and 7 of these parts called, and how many eighths are in a whole or one? *Ans:*

Solution—2 eighths, 3 eighths, 4 eighths, 5 eighths, 6 eighths, 7 eighths;

3. What is one eighth of 24? 48? 72? 88? *Ans:* 3; 6; 9; 11.

4. What is 2 eighths of 32? 40? 56? 72? *Ans:* 8; 10; 14; 18.

Solution—One eighth of 32 is 4. So 2 eighths is 2 times 4 or 8.

5. What is 3 eighths of 16? 64? 80? 32? *Ans:* 6; 24; 30; 12.

6. What is 5 eighths of 8? 24? 48? 64? *Ans:* 5; 15; 30; 40.

7. One eighth of 24 is how many times 3? *Ans:* One eighth of 24 is 3 which is 1 times 3.

8 Three eighths of 40 is how many times 5? *Ans:* One eighth of 40 is 5 which is 3 times 5.

9 Four eighths of 80 is how many times 8? *Ans:* One eighth of 80 is 10. 4 eighths is 4 times that, or 40, which is 5 times 8.

10. 5 eighths of 56 is how many times 7?

Solution—One eighth of 56 is 7. 5 eighths is 5 times that, or 35, which is 5 times 7.

11. Six eighths of 64 is how many times 12? *Ans:* One eighth of 64 is 8. 6 eighths is 8 times that, or 48, which is 4 times 12.

12. Seven eighths of 72 is how many times 3? *Ans:* 21.

13 Three eighths of 32 is how many times one third of 12? *Ans:* One eighth of 32 is 4, and 3 eighths is 3 times 4, or 12. One third of 12 is 4; The first part is 12, the second part of the problem is 4. 12 is 3 times 4.

14. 6 eighths of 40 is how many times one fourth of 24? *Ans:* One eighth of 40 is 5, and 6 eighths is 6 times 5, or 30; one fourth of 24 is 6; 30 is 5 times 6.

15 Four eighths of 48 is how many times 2 thirds of 18? *Ans:* 2.

16. Seven eighths of 96 is how many times 3 fifths of 10? *Ans:* 14.

17. 5 eighths of 56 is how many times 5 sixths of 42? *Ans:* 1.

18. 2 thirds of 27 is how many times 3 fourths of 12? *Ans:* 2.

19. If a single thing is divided into 9 equal parts, what are 1, 2, 3, 4, etc., of these parts called? *Ans:* One ninth, 2 ninths, 3 ninths, 4 ninths, etc.

20. What is 2 ninths of 18? 27? 45? 36? *Ans:* 4; 6; 10; 8.

21. What is 3 ninths of 63? 72? 81? 27? *Ans:* 21; 24; 27; 9.

22. What is 4 ninths of 9? 36? 54? 81? *Ans:* 4; 16; 24; 86.

23. What is 5 ninths of 54? 72? 63? 27? *Ans:* 30; 40; 35; 15.

24. What is 6 ninths of 81? 18? 36? 90? *Ans:* 54; 12; 24; 60.

25. What is 7 ninths of 18? 99? 27? 108? *Ans:* 14; 77; 21; 84.

26 Three times 6, plus 2 thirds of 6, is how many? *Ans:* 3 times 6 is 18; one third of 6 is 2, and 2 thirds of 6 is 2 times 2, or 4; 18 and 4 is 22.

27 Four times 12, plus 3 fourths of 12, is how many? *Ans:* 4 times 12 is 48; 3 fourths of 12 is

9; $48 + 9 = 57$.

28. 5 times 10, plus 3 fifths of 10, is how many? *Ans:* 56.

29. Six times 12, plus 3 sixths of 12, is how many? *Ans:* 78.

30. 5 times 7, plus 4 sevenths of 7, is how many? *Ans:* 39.

31. Nine times 8, plus 5 eighths of 8, is how many? *Ans:* 77.

32. 2 times 18, plus 7 ninths of 18, is how many? *Ans:* 50.

33. 2 ninths of 18 is how many times 2 thirds of 3?

Solution—2 ninths of 18 is 4; 2 thirds of 3 is 2; 4 is as many times 2 as 2 is contained in 4, or 2.

34. 5 ninths of 27 is how many times 5 sixths of 6? *Ans:* 5 ninths of 27 is 15; 5 sixths of 6 is 5; 15 is as many times 5 as 5 is contained in 15, or 3.

35. Six ninths of 54 is how many times 4 fifths of 15? *Ans:* 3.

36 Three ninths of 72 is how many times 2 eighths of 16? *Ans:* 6.

37. Seven eighths of 24 is how many times 7 eighths of 8? *Ans:* 3.

38. Kevin bought 15 new songs for his player, and deleted 6 of them, and then had 4 less than 20. How many songs did he have at first? *Ans:* If Kevin bought 15 new songs and deleted 6 of them, so from the new ones, he had $15 - 6$, or 9 songs left; but he had altogether $20 - 4$, or 16. So he had $16 - 9$, or 7 songs on his player.

39. Hill and Oliver each had 26 CDs; after Hill had given Oliver 10 CDs, and Oliver had given Hill 6 CDs. How many do they each have now? *Ans:* If Hill gave Oliver 10 CDs, and Oliver gave Hill 6 CDs, Oliver had $10 - 6$, or 4 CDs more than before, and Hill had 4 less than before. So Oliver had $26 + 4$, or 30, and Hill had $26 - 4$, or 22 CDs.

40. A grocer had 48 baskets of oranges, sold 2 fourths of them to one store, and one fourth to another. How many baskets did he sell to each? *Ans:* 24 baskets to the first; 12 baskets to the second.

41. Lucy bought 60 DVDs, and sold one third of them to Bob, and 3 fifths of the remaining to Carl. How many DVDs does she have now? *Ans:* Lucy sold Bob one third of 60 DVDs, or 20 DVDs, and had $60 - 20$, or 40 left; 3 fifths of 40 is 24. So she then has left $40 - 24$, or 16 DVDs.

42. 2 fourths is equal to what else? *Ans:* One half.

43. 4 sixths is equal to what else? *Ans:* 2 thirds.

44. What is 5 ninths of 18? *Ans:* 10.

45. What 2 thirds of 2 thirds of 9? *Ans:* 4.

46. What is 7 eighths plus 16 eighths? *Ans:* 23 eighths.

Lesson 4 - *Addition, Subtraction*

HARRY gave one third of an apple to his brother, and 2 thirds of the apple to his sister. How much did he give away?

Solution—3 thirds or all of it.

2. Matthew gave 2 fifths of a peach to Eliot, and 3 fifths to Morris. How much did he give to both? *Ans:* 5 fifths or all of it.

3. James gave 3 sevenths of a melon to Harry, and 4 sevenths to Harvey. How much did he give away? *Ans:* 7 sevenths or all of it.

4. Danny ate 3 eighths of a bag of cashews yesterday, and 4 eighths of the bag today. How much of the bag did he eat in all? *Ans:* 7 eighths.

5. Ella gave one fourth of a melon to Phoebe, 2 fourths to Carrie, and 3 fourths to Kate. How much did she give away? *Ans:* 6 fourths or $1\frac{1}{2}$.

6. Philip gave 2 sixths of a candy bar to Jane, 3 sixths to Sarah, and 5 sixths to Eliza. How much did he give away?

Ans: 10 sixths.

7. Willie lost 7 fifths of a dollar, but has 9 fifths of a dollar left. How much did he have at first?

Solution—If Willie lost 7 fifths of a dollar and had 9 fifths left, before his loss he had $7 + 9$, or 16 fifths of a dollar.

8. Matthew lost 6 eighths of his coins from one pocket, and 7 eighths from the other, and had 5 eighths left. How many did he have at first?

Solution—If he lost $6\text{ eighths} + 7\text{ eighths}$, which is 13 eighths of the coins, and has 5 eighths left. So he had $13 + 5$, or 18 eighths of coins.

9. Dora gave 3 ninths of a pound of raisins to Ella, and 7 ninths to Daisy, and then she had 3 ninths left. How many did she have at first? *Ans:* 13 ninths.

10. Jane had 7 eighths of a pound of candies, and gave Maria 5 eighths of a pound. How many eighths are left? *Ans:* 2 eighths.

11. Frank took 6 sevenths of a melon, and gave Drew 4 sevenths of the melon. How much is left? *Ans:* 2 sevenths of the melon.

12. Of her, 10 eighths quarter, Louisa gave Lizzie 7 eighths. How much is left? *Ans:* 3 eighths.

13. What is the difference between the sum of 4 sevenths and 6 sevenths and 5 sevenths? *Ans:* 5 sevenths.

14. Sallie had 24 pears, and gave Ben 2 eighths and Amanda 3 eighths of them. How many are left? *Ans:* 9 pears.

15. Jake had one third of a cup of peanuts, bought 4 thirds of a cup more, and then threw away 1 cup. What part of a cup is left?

Solution—One third + 4 thirds is 5 thirds; 1 cup contains 3 thirds; 5 thirds - 3 thirds is 2 thirds of a cup, left.

16. Peter had 5 sixths of a basket of apples, then he sold 3 sixths, and then bought 2 sixths of a basket, how many sixths does he have now?

Ans: 5 sixths - 3 sixths is 2 sixths; 2 sixths + 2 sixths is 4 sixths.

17. What is the difference between the sum of 3 eighths and 7 eighths, and the sum of 4 eighths

and 5 eighths? *Ans:* One eighth.

18. A shop bought 20 tires, and sold 2 tenths of them to customer A, 3 tenths to customer Bill and 4 tenths to customer C. How many tires are left? *Ans:* 2 tenths + 3 tenths + 4 tenths are 9 tenths; they had left 10 tenths - 9 tenths, or one tenth of 20 tires, or 2 tires.

19. A woman had 36 meters of tape. She gave 5 ninths it to one person, and 3 ninths of it to another. How much does she have now?

Solution—5 ninths + 3 ninths is 8 ninths. So she had left 9 ninths - 8 ninths, or one ninth of 36 meters, or 4 meters.

20. Mariana had 3 fourths of a pint of nuts, Elva had twice as many, and Ezra 3 times as many. How many did they all have? *Ans:* 18 fourths.

21. A bought 4 ninths of a bag of wheat, and Bill bought 3 times as much. How much did Bill buy? *Ans:* 12 ninths.

22. At 7 fifths of a dollar each, how much will 5 candies cost? *Ans:* 35 fifths, or \$7.

23. Mary had 1 and one fifths of a melon, gave 2 fifths of it to Sarah, and twice as much to Sophia. How much is left?

Solution—Sarah received 2 fifths and Sophia received twice 2 fifths, or 4 fifths; both receive 2 fifths + 4 fifths, or 6 fifths of a melon; 6 fifths minus one fifth is 5 fifths, or 1 melon.

24. Cornell gave 3 times 3 sixths of an apple to Gray, and had 4 times 3 sixths left. How much did he have at first? *Ans:* 21 one sixths.

25. What will one fifth of a meter of tape cost, at the rate of 20 fourths dollars a meter? What will 10 fifths cost at the same rate?

Ans: If it is 20 fourths dollar for a meter, then one fifth will cost 4 fourths. 4 fourths is one dollar. If it is one dollar for one fifth, then it is 10 fifths dollar or \$2.

26. Stanton had 2 thirds of a dollar, found one half of 4 thirds of a dollar. How many thirds of a dollar did he have then? *Ans:* 4 thirds.

27. Michael bought 7 tenths of a barrel of monkeys, and then sold 2 thirds of 6 tenths of a barrel. How much is left? *Ans:* 3 tenths of a barrel.

28. Thornton had 8 sixths of a box of paper, bought 3 fourths of 20 sixths of a box. How much does he have now? *Ans:* 23 sixths of a box.

29. Ferris lost 6 ninths of a dollar, and then, found 3 ninths of a dollar, and now has 3 fourths of 8 ninths of a dollar left. How much did he have at first? *Ans:* 1 dollar.

30. What is one thirds of 6 sevenths? *Ans:* 2 sevenths.

31. What is 2 thirds of 9 tenths? *Ans:* 6 tenths.

32. What is 3 fifths of 10 twelveths? *Ans:* one half.

33. What is one fifth of 10 thirteenth? *Ans:* 2 thirteenths.

34. What is 4 fifths of 10 fifteenths? *Ans:* 8 fifteenths.

35. What is one eights of 2 thirds of 12? *Ans:* 1.

36. What is 3 halves of 3 tenths of 4? *Ans:* 9 fifths.

37. What is 4 thirds of one fifth of 15? *Ans:* 4.

38. What is 6 thirds of one third of 9? *Ans:* 6.

39. What is the sum of one half and one half? *Ans:* 1.

40. What is the sum of one third and one third? *Ans:* Two thirds.

50. What is the sum of one fourth and one fourth? *Ans:* One half.

51. If you take a pie and cut two pieces out of it, a one half piece and a one third piece. Which is bigger, one half or one third? *Ans:* One half.

52. If you take a pie and cut two pieces out of it, a one third piece and a one fourth piece. Which piece is bigger, one fourth or one third? *Ans:* One third.

Lesson 6 - *Number analysis*

HOW much will one meter of wire cost, if 2 thirds of a meter costs \$4?

Solution—If 2 thirds of a meter of wire cost \$4, one third of a meter will cost one half of \$4, or \$2, and 3 thirds, or one meter, will cost 3 times \$2, which is \$6.

2. What will one box of soap cost, if 3 fourths of a box costs 3 fourths of a dollar? *Ans:* One dollar.

3. What will one box of erasers cost, if 2 thirds of a box costs \$2? *Ans:* \$3.

4. What will one box of paper cost, if 3 fourths of a box costs \$6?

Ans: If 3 fourths of a box of soap cost \$6, one fourth of a box will cost one third of \$6, or \$2, and 4 fourths, or one box, will cost 4 times \$2, or \$8.

4. If 3 fifths of a ticket costs \$9, what will one ticket cost?

Ans: If 3 fifths of a ticket costs \$9, one fifth of a ticket will cost one third of \$9, or \$3, and 5 fifths, or one ticket, will cost 5 times \$3, or \$15.

5. If 3 fourths of a cake costs \$12, what will the whole cake cost?

Ans: If 3 fourths of a cake costs \$12, then one fourth the cake will cost one fourth of \$12, or \$4, and 4 fourths, or the whole cake, will cost 4 times \$4, or \$16.

6. If 4 fifths of a box of tea cost \$8, what will one box cost?

Ans: If 4 fifths of a box cost \$8, one fifth of a box will cost one fourth of \$8, or \$2, and 5 fifths, or one box, will cost 5 times \$2, or \$10.

7. If 3 fifths of a meter of speaker wire cost \$6, what will one meter cost?

Ans: If 3 fifths of a meter cost \$6, one fifth of a meter will cost one third of \$6, or \$2, and 5 fifths, or 1 meter, will cost 5 times \$2, or \$10.

8. What will two pounds of detergent cost, if 5 sixths of a pound cost 10 cents?

Ans: If 5 sixths of a pound cost 10 cents, one sixth of a pound will cost one fifth of 10 cents, or 2 cents, and 6 sixths, or 1 pound, will cost 6 times 2 cents, or 12 cents. And 2 pounds will cost 2 times 12, or 24 cents.

9. What is the cost of 2 bags of styrofoam, at the rate of \$4 for 4 sixths of a bag?

Ans: If 4 sixths of a bag cost \$4, one sixth of a bag will cost one fourth of \$4, or 1 dollar, 6 sixths, or 1 bag, will cost \$6, and 2 bags will cost 2 times \$6, or \$12.

What is the cost of one of these, if

10. Two of them cost 2 sixths of a coin? *Ans:* Two twelfths or one sixth.

11. Three of them cost 4 fifths of a coin? *Ans:* Twelve fifths.

12. Fifteen of them cost 5 thirds of a coin? *Ans:* 25 coins.

13. What is the cost of 3 meters of cloth if 3 sevenths of a meter costs \$6? *Ans:* \$42.

14. How far can Alan walk in 4 days, if in 5 sixths of a day he can walk 20 miles? *Ans:* 96 miles.

15. What is the cost of 5 pounds of steak, if 3 fifths of a pound cost \$6? *Ans:* \$50.

16. What is 5 times the distance to Lancaster, if 3 fourths of the distance is 3 miles? *Ans:* 20 miles.

17. How much will 4 bags of apples cost, if 5 tenths of a bag cost 50 cents? *Ans:* 400 cents.

Lesson 19 - *Subtraction of Fractions*

WHAT is the difference between $\frac{6}{7}$ and $\frac{3}{7}$?

Solution— $\frac{6}{7} - \frac{3}{7} = \frac{4}{7}$.

2. What is the difference between $\frac{7}{8}$ and $\frac{3}{8}$? *Ans:* $\frac{7}{8} - \frac{3}{8} = \frac{4}{8}$, or $\frac{1}{2}$.

3. What is the difference between $\frac{8}{9}$ and $\frac{5}{9}$? *Ans:* $\frac{8}{9} - \frac{5}{9} = \frac{3}{9}$, or $\frac{1}{3}$.

4. What is the difference between $2\frac{3}{4}$ and $1\frac{1}{4}$?

Solution— $2 - 1 = 1$; $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$, or $\frac{1}{2}$. The whole number difference is $1\frac{1}{2}$. Of course we could have done this problem by first reducing the mixed fraction to an improper fraction and then doing the subtraction. But often you can skip that step and do the addition or the subtraction by first carefully looking at the mixed fraction to see if they can be added or subtracted directly as in this case.

5. What is the difference between $3\frac{2}{3}$ and $2\frac{1}{3}$? *Ans:* $1\frac{1}{3}$.

6. What is the difference between $5\frac{3}{5}$ and $2\frac{2}{5}$? *Ans:* $3\frac{1}{5}$.

7. How many fourths in $\frac{1}{2}$ minus $\frac{1}{4}$? *Ans:* $\frac{1}{4}$.

8. How many eighths in $\frac{7}{8}$ minus $\frac{3}{4}$? *Ans:* $\frac{1}{8}$.

9. How many sixths in $\frac{1}{2}$ minus $\frac{1}{3}$? *Ans:* $\frac{1}{6}$.

10. How many twelfths in $\frac{5}{6}$ minus $\frac{3}{4}$? *Ans:* $\frac{1}{12}$.

Subtract

11. $\frac{1}{3}$ from $\frac{1}{2}$.

Solution—1 is equal to $\frac{3}{6}$, and $\frac{1}{3}$ to $\frac{2}{6}$; $\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$.

12. $\frac{1}{5}$ from $\frac{1}{4}$.

Solution— $\frac{1}{5}$ is equal to $\frac{4}{20}$, and $\frac{1}{4}$ to $\frac{5}{20}$; $\frac{5}{20} - \frac{4}{20} = \frac{1}{20}$.

13. $\frac{2}{3}$ from $\frac{3}{4}$.

Ans: $\frac{2}{3}$ is equal to $\frac{8}{12}$, and $\frac{3}{4}$ to $\frac{9}{12}$; $\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$.

14. $\frac{1}{6}$ from $\frac{1}{4}$ *Ans:* $\frac{1}{12}$.

15. $\frac{2}{5}$ from $\frac{2}{4}$ *Ans:* $\frac{1}{10}$.

16. $\frac{2}{3}$ from $\frac{4}{5}$ *Ans:* $\frac{2}{15}$.

17. $\frac{2}{5}$ from $\frac{1}{10}$ *Ans:* $\frac{3}{10}$.

18. $\frac{3}{5}$ from $\frac{3}{4}$ *Ans:* $\frac{3}{20}$.

19. $\frac{3}{4}$ from $\frac{8}{9}$ *Ans:* $\frac{5}{36}$.

20. $\frac{1}{6}$ from $\frac{1}{5}$ *Ans:* $\frac{1}{20}$.

21. $\frac{1}{7}$ from $\frac{1}{6}$ *Ans:* $\frac{1}{42}$.

22. $\frac{1}{3}$ from $\frac{5}{8}$ *Ans:* $\frac{7}{24}$.

23. $\frac{2}{5}$ from $\frac{2}{3}$ *Ans:* $\frac{4}{15}$.

24. $\frac{1}{2}$ from $\frac{5}{7}$ *Ans:* $\frac{3}{14}$.

25. $\frac{1}{8}$ from $\frac{1}{7}$ *Ans:* $\frac{1}{56}$.

26. $\frac{1}{9}$ from $\frac{1}{2}$ *Ans:* $\frac{7}{18}$.

27. $\frac{2}{7}$ from $\frac{2}{3}$ *Ans:* $\frac{8}{21}$.

28. $\frac{3}{7}$ from $\frac{6}{8}$ *Ans:* $\frac{9}{28}$.

29. $2\frac{1}{2}$ from $3\frac{1}{4}$ *Ans:* $\frac{3}{4}$.

30. $3\frac{1}{5}$ from $4\frac{1}{4}$ *Ans:* $1\frac{1}{20}$.

31. $2\frac{1}{4}$ from $3\frac{1}{6}$ *Ans:* $1\frac{1}{12}$.

32. $3\frac{1}{5}$ from $5\frac{1}{6}$ *Ans:* $1\frac{17}{30}$.

33. A man owned $\frac{3}{4}$ of a boat, and sold $\frac{1}{3}$ of the boat. What part of the boat does he still own?

Solution—If he owned $\frac{3}{4}$ of a boat and sold $\frac{1}{3}$ of a boat, he kept the difference between $\frac{3}{4}$ and $\frac{1}{3}$; $\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{3} = \frac{4}{12}$; $\frac{9}{12} - \frac{4}{12} = \frac{5}{12}$, the share he still owned.

34. A pole is in the mud, air, and water. If $\frac{3}{5}$ of it is in the mud and water, how much is in the air?

Ans: If $\frac{3}{5}$ is in the mud and water, $\frac{5}{5} - \frac{3}{5}$, or $\frac{2}{5}$, is the part in the air.

35. If $\frac{1}{2}$ of a pole is in the air, $\frac{1}{3}$ in the water, and the rest in the mud, how much is in the mud?

Ans: $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$, the part in the air-and water; $\frac{6}{6} - \frac{5}{6} = \frac{1}{6}$, the part in the mud.

36. Michael Wilson's age, reduced by its $\frac{1}{4}$, is 15 years. How old is he?

Ans: $\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$. If $\frac{3}{4}$ of her age is 15 years, his age is 20 years.

37. The difference between $\frac{2}{3}$ of my money and $\frac{3}{4}$ of my money is \$90. How much money do I have?

Ans: $\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$. If $\frac{1}{12}$ of my money is \$9, $\frac{11}{12}$ will be \$108.

38. Mr. Smith bought 50 meters of a sail cloth, and used just $\frac{3}{5}$ of it. How many meters does he have left?

Ans: $\frac{5}{5} - \frac{3}{5} = \frac{2}{5}$; $\frac{2}{5}$ of 50 meters are 20 meters, the amount left.

39. Nina owes $\frac{3}{5}$ of a dollar for a purchase. If she hands the clerk $\frac{3}{4}$ of a dollar, how much change should she receive?

Ans: $\frac{3}{4}$ of a dollar, minus $\frac{3}{5}$ of a dollar, equals $\frac{3}{20}$ of a dollar, or 15 cents.

40. In an orchard $\frac{1}{2}$ of the trees are apples, $\frac{1}{3}$ are peaches, and the rest are pear trees. What part of the whole are pear trees?

Ans: $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$; $\frac{6}{6} - \frac{5}{6} = \frac{1}{6}$, the number of pear trees.

41. A boy had 36 marbles, lost $\frac{3}{4}$ of them, and then found $\frac{5}{6}$ as many as he had at first. How many did he have then?

Ans: $\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$; $\frac{1}{4}$ of 36 marbles is 9 marbles, the number left; $\frac{5}{6}$ of 36 is 30; 30 marbles + 9 marbles = 39 marbles.

42. \$40 is 4 times what Aron paid for parking, and the cost of the show is $\frac{1}{8}$ of the cost of parking. What is the cost of the show?

The cost of the parking is $\frac{1}{4}$ of \$40, or \$10, which is $\frac{1}{8}$ of the cost of the show. So the show costs 8 times \$10, or \$80.

What is the value of? The following problems are encountered often so it is preferred that child memorize the results. Have child do these until, he can answer them immediately.

43. $\frac{1}{2} + \frac{1}{3}$ *Ans:* $\frac{5}{6}$.

44. $\frac{1}{2} + \frac{1}{4}$ *Ans:* $\frac{3}{4}$.

45. $\frac{1}{2} + \frac{1}{6}$ *Ans:* $\frac{2}{3}$.

46. $\frac{1}{3} + \frac{1}{6}$ *Ans:* $\frac{1}{2}$.

47. $\frac{1}{3} + \frac{1}{4}$ *Ans:* $\frac{7}{12}$.

48. $\frac{1}{4} + \frac{1}{8}$ *Ans:* $\frac{3}{8}$.

49. $\frac{1}{4} + \frac{1}{5}$ *Ans:* $\frac{9}{20}$.

50. $\frac{1}{4} + \frac{2}{3}$ *Ans:* $\frac{11}{12}$.

51. $\frac{2}{3} + \frac{1}{2}$ *Ans:* $\frac{7}{6}$.

52. $\frac{3}{4} + \frac{2}{3}$ *Ans:* $\frac{3}{12}$.

53. $(\frac{1}{2} + \frac{1}{3}) - \frac{1}{6}$ *Ans:* $\frac{2}{3}$.

54. $(\frac{1}{2} + \frac{1}{3}) - \frac{1}{4}$ *Ans:* $\frac{11}{12}$.

55. $(\frac{1}{3} + \frac{1}{6}) - \frac{1}{2}$ *Ans:* $\frac{3}{20}$.

56. $(\frac{1}{2} + \frac{3}{4}) - \frac{1}{3}$ *Ans:* $\frac{5}{12}$.

57. $(2\frac{1}{2} + 4\frac{1}{2}) - 4\frac{1}{4}$ *Ans:* $2\frac{3}{4}$.

58. $(\frac{1}{4} + \frac{1}{5}) - \frac{1}{5}$ *Ans:* $\frac{1}{4}$.

59. $(1\frac{1}{2} + 1\frac{1}{4}) - 2\frac{1}{2}$ *Ans:* $\frac{1}{4}$.

60. $(3\frac{2}{3} + 2\frac{3}{4}) - 5\frac{1}{12}$ *Ans:* $\frac{1}{6}$.

61. \$600 is $\frac{3}{5}$ of what Ron paid for a computer, and the cost of the computer, increased by its 3 fifths, is 5 times what he paid for the software. What is the cost of the software.

Ans: If $\frac{3}{5}$ of the cost of the computer is \$600, the whole cost if the computer is \$1000. $\$1000 + \$600 = \$1600$, which is 5 times the cost of the software. So the cost of the software is $\frac{1}{5}$ of \$1600, or \$320.

Lesson 20 - Compound to Simple Fractions

WHAT is $\frac{1}{3}$ of 4?

Solution— $\frac{1}{3}$ of 1 is $\frac{1}{3}$, and if $\frac{1}{3}$ of 1 is $\frac{1}{3}$ then $\frac{1}{3}$ of 4 is 4 times $\frac{1}{3}$ which is $\frac{4}{3}$. Therefore $\frac{1}{3}$ of 4 is $\frac{4}{3}$ of one.

2. What is $\frac{1}{3}$ of 5? $\frac{1}{4}$ of 6? *Ans:* $\frac{5}{3}$; $\frac{6}{4}$.

3. What is $\frac{1}{5}$ of 7? $\frac{1}{6}$ of 9? *Ans:* $\frac{7}{5}$; $\frac{9}{6}$.

4. What is $\frac{1}{2}$ of 5? $\frac{1}{4}$ of 10? *Ans:* $\frac{5}{2}$; $\frac{10}{4}$.

5. What is $\frac{1}{7}$ of 9? $\frac{1}{8}$ of 20? *Ans:* $\frac{9}{7}$; $\frac{20}{8}$.

6. What is $\frac{2}{4}$ of 6? $\frac{3}{4}$ of 10? *Ans:* $\frac{12}{4}$ or 3; $\frac{30}{4}$ or $\frac{15}{2}$.

7. What is $\frac{3}{4}$ of 3? $\frac{5}{6}$ of 14? *Ans:* $\frac{9}{4}$; $\frac{70}{6}$.

8. What is $\frac{2}{7}$ of 4? $\frac{3}{9}$ of 15? *Ans:* $\frac{8}{7}$; $\frac{45}{9}$.

9. Mary has \$6, and her brother has $\frac{3}{5}$ as much. How much does her brother have?

Ans: $\frac{1}{5}$ of 6 is $\frac{6}{5}$, and $\frac{3}{5}$ is 3 times $\frac{6}{5}$, or $\frac{18}{5}$, or $\$3\frac{3}{5}$.

10. Asha has \$20, and $\frac{1}{3}$ of her money equals 4 times Bina's money. How much money does Bina have?

Ans: $\frac{1}{3}$ of 20 is $\frac{20}{3}$. If $\frac{20}{3}$ is 4 times Bina's money, then Bina's money is $\frac{1}{4}$ of $\frac{20}{3}$, or $\frac{5}{3}$ of \$20.

11. James is 7 years old, and $\frac{4}{9}$ of his age equals $\frac{7}{9}$ of Mary's age. How old is Mary?

Solution— $\frac{4}{9}$ of 7 years is $\frac{28}{9}$ years, which are $\frac{7}{9}$ of Mary's age. So $\frac{1}{9}$ of Mary's age equals $\frac{1}{7}$ of $\frac{28}{9}$, $\frac{4}{9}$ years and her age equals 9 times $\frac{4}{9}$ years, or 4 years.

12. A movie tickets cost \$21 to the family. $\frac{4}{5}$ of the tickets cost is $\frac{7}{10}$ of the cost of the snacks they bought. What is the cost of both tickets and the snacks?

Ans: $\frac{4}{5}$ of \$21 is $\$8\frac{4}{5}$. If $\frac{8\frac{4}{5}}$ is $\frac{7}{10}$ of the cost of the movie, then $\frac{1}{10}$ of the cost is $\frac{1}{7}$ of $\frac{8\frac{4}{5}}$ or $\frac{12}{5}$. 10 times $\frac{12}{5}$ is \$24.

13. A jug contains 5 gallons, and $\frac{6}{7}$ of its contents is $\frac{10}{14}$ the contents of another jug. What is the content in gallons of the second jug?

Ans: $\frac{6}{7}$ of 5 gallons are $\frac{30}{7}$ gallons. If $\frac{30}{7}$ gallons is $\frac{10}{14}$ of the second jug, $\frac{1}{14}$ of the v is $\frac{1}{10}$ of $\frac{30}{7}$ or $\frac{3}{7}$ of a gallon and $1\frac{4}{14}$ is 6 gallons.

14. What is $\frac{2}{3}$ of $\frac{6}{8}$?

Solution— $\frac{1}{3}$ of $\frac{6}{8}$ is $\frac{6}{24}$, and if $\frac{1}{3}$ of $\frac{5}{8}$ is $\frac{6}{24}$, then $\frac{2}{3}$ of $\frac{6}{8}$ is 2 times $\frac{6}{7}$, which is $\frac{4}{8}$, or $\frac{1}{2}$.

What is?

15. What is $\frac{2}{3}$ of 40? *Ans:* 24.

16. What is $\frac{5}{4}$ of 36? *Ans:* 45.

17. What is $\frac{4}{5}$ of 60? *Ans:* 49.

18. What is $\frac{1}{2}$ of $\frac{6}{9}$? *Ans:* $\frac{1}{3}$.

19. What is $\frac{1}{3}$ of $\frac{4}{5}$? *Ans:* $\frac{4}{15}$.

20. What is $\frac{1}{4}$ of $4\frac{1}{2}$? *Ans:* $1\frac{1}{8}$.

21. What is $\frac{1}{5}$ of $2\frac{1}{2}$? *Ans:* $\frac{1}{2}$.

22. What is $\frac{1}{3}$ of $3\frac{1}{3}$? *Ans:* $1\frac{1}{9}$. Note that one third of 3 is 1 and one third of one one third is one ninth. By carefully looking at the numbers, often we can solve the problem using much easier method.

What is?

Note—The term "of" between two numbers often means the product of, or multiplication. So if we say what is of $\frac{2}{3}$ of 3, then it means we can multiply the first term with the second one.

23. $\frac{2}{3}$ of $\frac{6}{7}$?

Ans: $\frac{1}{3}$ of $\frac{6}{7}$ is $\frac{2}{7}$, and $\frac{2}{3}$ of $\frac{6}{7}$ is 2 times $\frac{2}{7}$, or $\frac{4}{7}$. Or we could have multiplied $\frac{2}{3}$ with $\frac{6}{7}$, canceling 3 with 6, $\frac{2}{1}$ with $\frac{2}{7}$, we get $\frac{4}{7}$.

24. $\frac{2}{7}$ of $\frac{7}{8}$?

Ans: $\frac{1}{7}$ of $\frac{7}{8}$ is $\frac{1}{8}$, and $\frac{2}{7}$ of $\frac{7}{8}$ is $\frac{2}{8}$, or $\frac{1}{4}$.

25. $\frac{3}{6}$ of $\frac{6}{9}$? *Ans:* $\frac{1}{3}$.

26. $\frac{4}{9}$ of $\frac{18}{20}$? *Ans:* $\frac{2}{5}$.

27. $\frac{3}{5}$ of $\frac{15}{18}$? *Ans:* $\frac{1}{2}$.

28. $\frac{2}{7}$ of $\frac{14}{16}$? *Ans:* $\frac{1}{4}$.

29. $\frac{3}{5}$ of $2\frac{1}{2}$? *Ans:* $\frac{3}{2}$.

30. $\frac{5}{6}$ of $2\frac{2}{5}$? *Ans:* 2.

31. $\frac{3}{5}$ of 6 *Ans:* $2\frac{3}{5}$.

32. $\frac{2}{5}$ of $3\frac{3}{4}$? *Ans:* $\frac{3}{2}$.

33. $\frac{3}{2}$ of 11? *Ans:* $16\frac{1}{2}$.

34. $\frac{3}{5}$ of \$40 is 2 times what a paid for a calendar. What was the cost of the calendar? *Ans:* \$12.

28. $\frac{5}{7}$ of $\frac{14}{25}$ of \$100 is $\frac{1}{10}$ the cost of a watch. How much did the watch cost? *Ans:* \$400.

29. Lim has 27 points, and $\frac{2}{3}$ of Lim's points equals $\frac{2}{5}$ of Chee's number. How many marbles has Chee? *Ans:* 45 points.

30. Ivan's hat cost $\frac{4}{5}$ of an \$20, which is $\frac{5}{3}$ of $\frac{3}{5}$ of the cost of his coat. What is the cost of his shirt. *Ans:* \$16.

31. Out of 48 chestnuts, how many does each of two boys receive, if a receives $\frac{6}{8}$, and Bill receives $\frac{2}{3}$ as many as A *Ans:* A, 36 chestnuts; B, 24.

32. Hanah bought $\frac{4}{6}$ of a box of clips, which is $\frac{8}{3}$ of what Sarah bought. How many clips did each purchase; provided there were 24 clips in box? *Ans:* Hanah, 16 ; Sarah, 2.

33. A has 40 fruit trees, $\frac{4}{10}$ of which are apples, $\frac{1}{2}$ of the rest are pears, and the rest are peaches. How many trees are there of each kind? *Ans:* 16 apple, 12 pear, 12 peach.

34. Jenson's age, reduced by its $\frac{1}{4}$ and $\frac{1}{5}$, is 20 years, and his age is $\frac{4}{5}$ of his uncle's age. What is the age of each?

Ans: $\frac{1}{4} + \frac{1}{5} = \frac{9}{20}$; $\frac{20}{20} - \frac{9}{20} = 1\frac{1}{20}$. If $\frac{11}{20}$ of his age is 22 years, $\frac{20}{20}$ is 40 years. If $\frac{4}{5}$ of his uncle's age is 40 years, his uncle's age is 50 years.

Lesson 31 - *One part a given number more than another*

Alan and Bob have 25 oranges. How many each has, if Bob has 5 more than Alan?

Solution—Alan's number plus 5 equals Bob's number. Alan's number is also equal to 25 minus Bob's number. Five plus Bob's number is equal to 25 minus Bob's number. So twice Bob's number is equal to 20, So Bob's number is 10 and Alan's number is 15.

Another way to do that is to subtract 5 from 25, we get 20. Divide 20 by 2 and 10. Add 5 for Alan, we get 15 for Bob and 10 for Alan.

2. Marina has seven oranges more than Andre, and together they have 27. How many do they each have?

Ans: The difference between the two numbers is 7. So subtract 7 from 27 and we get 20. We give 10 to each, and then give Marina the 7 that we subtracted. Marina, 17 oranges. Andre, 10 oranges.

3. Stephen has \$10 more than Martha, and together they have \$40. How much does each have?

Ans: First subtract the difference of 10 from 40, then divide 30 by 2 and add 10 for the larger part. Stephen \$25; Martha, \$15.

4. The sum of two numbers is 31, and their difference 5. What are the numbers? *Ans:* 13, 18.

5. The sum of two numbers is 65, and their difference 11. What are the numbers? *Ans:* 27, 38.

6. The sum of two numbers is 56, and their difference 40. What are the numbers? *Ans:* 8, 48.

7. Divide number 28 into two parts, such that one part is 6 less than the other. *Ans:* 11, 17.

8. Thomas and Reuben each have the same amount of money. Reuben found \$9, and then together they had \$45. How much did each have?

Ans: Before Reuben found \$9, they had \$45 - \$9 = \$36. So each at first had $\frac{1}{2}$ of \$36, or \$18.

9. Ellen and Kate each had the same number of candies; Ellen ate 5 of hers, and now together they have 21 left. How many did each have at first?

Ans: Before Ellen ate 5 candies, they had $21 + 5 = 26$. So each at first had $\frac{1}{2}$ of 26, or 13 candies.

10. Two boys found an equal number of green rocks at the beach; one threw away 6 and the other 4, and together they kept 22. How many did they each find?

Ans: Together they both threw away 10. So before throwing away any, they had $22 + 10$, or 32. And each, therefore, found $\frac{1}{2}$ of 32, or 16 rocks each.

11. Ahmad and Ben had equal sums of money; Ahmad lost \$5, and Ben gained \$7, and then together they had \$36 left. How much did each have at first?

Ans: They now have \$7 - \$5, or \$2 more than at first. So they had at first \$34, and each had \$17.

12. Daniel and Edwin each had the same number of shells; Daniel lost 6, and Edwin gave him 4 of his, and now together they have 14 shells. How many had each before?

Ans: Daniel lost 6, and Edwin gave him 4 of his, so he has 2 less than he had before. Divide 14 minus 2, 12

13. Olga and Ivan each had the same number of pebbles; Olga threw away 10, and Ivan gave her 4 of his, and now together they have 24. How many did each have before?

Ans: They had 34 to start with or 17 each. Then Olga threw away 10 and got 4, so she now has 6 less or 11 and Ivan has 4 less 11 pebbles.

14. Three times Tommy's age, increased by 5 years equals Harvey's age, and the sum of their ages is 45 years. How old is each?

Ans: Three times Tommy's age, + 5 years, equals Harvey's age, which, added to Tommy's age, equals 4 times Tommy's, + 5 years, which is 45 years. If 4 times Tommy's age, + 5 years, equals 45 years, 4 times Tommy's age equals 45 years, - 5 years, or 40 years, and Tommy's age is 10 years, and Harvey's, 35.

15. Divide number 48 into two parts, such that twice the first part, reduced by 6, equals the second part.

Ans: 2 times the first part, minus 6, + the first part, or 8 times the first part, minus 6 = 48. So 8 times the first part = 48 + 6 = 54, the first part equals 18, and the second equals 30.

16. The sum of two numbers is 55. The larger number equals 3 times the smaller number, reduced by 5. What are the numbers?

Ans: 3 times the smaller number, minus 5, + the smaller number, or 4 times the smaller number, minus 5 = 55; 4 times the smaller number = 60, the smaller number equals 15, and the larger number 40.

17. A pole, whose length was 48 meters, was broken into two unequal pieces, such that $\frac{3}{5}$ of the longer piece equals the shorter piece. What is the length of each piece?

Ans: $\frac{5}{5}$ of the longer part + $\frac{3}{5}$ of the longer part, or $\frac{8}{5}$ of the longer part = 48 meters. So the two parts are 30 meters and 18 meters.

18. A locket and chain cost \$85. $\frac{3}{10}$ of the cost of the locket, plus \$7, equals the cost of the chain. What is the cost of each?

Ans: $\frac{10}{10}$ of the cost of the locket + $\frac{3}{10}$ of its cost, + \$7 = \$85. So $\frac{13}{10}$ of the cost of the locket = \$78. The locket cost \$60, and the chain \$25.

19. Francis has \$9 more than $\frac{1}{2}$ as much as Puja, and together they have \$42. How many dollars does each have?

Ans: $\frac{2}{2}$ of Puja's money + $\frac{1}{2}$ of Puja's money, + \$9 = \$42; $\frac{3}{2}$ of Puja's money = \$33. So Puja had \$22, and Francis \$20.

20. A drill and plane cost \$132. What is the cost of each, if the drill cost $\frac{2}{5}$ as much as the plane, minus \$8?

Ans: Drill cost \$100 and the plane \$32.

21. A tower, whose height was 45 meters, was measured in two unequal parts, and $\frac{3}{5}$ of the longer part, plus 5 meters, equals the shorter part. What is the length of each part?

Ans: Three-fifths of the longer part, + 5 meters, equals the shorter, which, added to the longer, equals $\frac{8}{5}$ of the longer, + 5 meters, which equals 45 meters. So $\frac{8}{5}$ of the longer = 40 meters, and the longer equals 25 meters, and the shorter, 20 meters.

22. A man walked 110 miles in three days. He walked 5 miles farther the second day than the first, and 10 miles farther the third day than the second. How far did he walk each day?

Ans: If he walked 5 miles further the second day than the first, and 10 miles further the third day than the second, then once the distance he walked the first day, + 5 miles, equals the distance he walked the second day, and once the distance he walked the first day, + 5 miles, + 10 miles, or once the distance he walked the first day, + 15 miles, equals the distance he walked the third

day. So 3 times the distance he walked the first day, + 5 miles + 15 miles = 110 miles: 3 times the distance - 90 miles; the distance walked the first day was 30 miles, the second day 35 miles, and the third day 45 miles.

23. A man bought a kayak, trailer, and oars for \$152; for the kayak he paid twice as much as for the oars, plus \$8, and for the trailer 4 times as much as for the oars, plus \$6. What did he pay for each?

Ans: One times the cost of the oars + twice the cost of the oars, + \$6 + 4 times the cost, + \$8, or 7 times the cost of the oars, + \$12 = \$152; 7 times the cost = \$140. So the oars cost \$20, the kayak \$46, and the trailer \$86.

24. There are 42 animals in a field, consisting of horses, sheep, and cows. What is the number of each provided $\frac{1}{2}$ of the number of sheep, + 10, equals the number of cows, and $\frac{1}{3}$ of the number of sheep, + 10, equals the number of horses.

Ans: $\frac{3}{6}$ of the number of sheep, + 10, which is the number of cows, + $\frac{2}{6}$ of the number of sheep, + 10, which is the number of horses, + $\frac{6}{6}$ of the number of sheep, equals $1\frac{1}{6}$ of the number of sheep, + 20, which equals 42. So $1\frac{1}{6}$ of the number of sheep equals $42 - 20 = 22$, and the number of sheep is 12, of cows 16, of horses 14.

25. There are 54 instruments in a group, consisting of horns, drums, and tuba. The number of tuba are $\frac{1}{2}$ of the drums, the number of drums are $\frac{1}{3}$ of horns. How many each are there? *Ans:* 36 Horns, 12 drums and 6 tuba.

Lesson 46 - More advanced problems

THE square of a certain number is 64. What is the number?

Solution—If the square of a number equals 64, the number equals the square root of 64, which is 8.

2. A boy when asked his age, replied, 3 times the square of my age equals 75 years. How old was he?

Ans: If 3 times the square of his age is 75 years, the square of his age equals $\frac{1}{3}$ of 75 years, or 25 years, and his age equals the square root of 25 years, or 5 years.

3. Albert, when asked how many marbles he had, answered, $\frac{1}{2}$ of the square of the number equals 18. How many marbles did he have?

Ans: The square of the number is 36. So the number is the square root of 36, or 6.

4. Three-fourths of the square of the number of letters in a sentence equals 27. How many letters are there in the sentence?

Ans: The square of the number is 36. So the number is 6.

5. The square of twice a number equals 256. What is that number, and what is the square of $\frac{1}{2}$ of the number?

Note—The square root of 256, which is 16, equals twice the number, etc. Or, the square of twice a number equals four times the square of the number, etc.

The square of twice a number $(2x)^2$, equals 4 times the square of the number, which equals 256. So the square of the number equals $\frac{1}{4}$ of 256, or 64, and the number equals the square root of 64, or 8; $\frac{1}{2}$ of the number is 4, and its square is 16.

6. If $\frac{2}{5}$ of the number of trees in an orchard is squared, the result will be 100. How many trees are there in the orchard?

Ans: If the square of $\frac{2}{5}$ of the number equals 100, $\frac{2}{5}$ of the number equals the square root of 100, or 10, and $\frac{2}{5}$ equals 25.

7. The square of twice a number is 18 more than twice the square of the number. What is the number?

Ans: The square of twice a number equals 4 times the square of the number; then 4 times the square, minus 2 times the square, or 2 times the square = 18. So the square of the number is 9, and the number is 3.

8. Twice the square of a number is 8 more than 6 times the square of half the number. What is the number?

Ans: 6 times the square of half a number, $6 \times \frac{1}{2} \times 2$, equals $\frac{6}{4}$ of the square of the number; then 2 times the square minus $\frac{6}{4}$ of the square, or $\frac{1}{2}$ of the square, equals 8; the square = 16, and the number is 4.

9. Three-fourths of the square of a number is 36 more than $\frac{3}{4}$ of the square of half the number. What is the number.

Ans: The square of $\frac{1}{2}$ the number equals $\frac{1}{4}$ of the square of the number, and $\frac{3}{4}$ of this is $\frac{3}{16}$ of the square of the number; then $\frac{3}{4}$ of the square, minus $\frac{3}{16}$ of the square, or $\frac{9}{16}$ of the square = 36. So the square = 64, and the number = 8.

10. Fifteen is 3 more than $1\frac{1}{2}$ times the cube of a number. What is that number?

Ans: $15 - 3 = 12$, 12 is $1\frac{1}{2}$ times bigger than 8. The cube of the number = 8. So the number is the cube root of 8, or 2.

11. Two-thirds of the cube of a number is 10 more than the cube of $\frac{2}{3}$ of the number. What is the number?

Ans: The cube of $\frac{2}{3}$ of the number equals $\frac{8}{27}$ of the cube of the number. So $\frac{2}{3}$, or $\frac{18}{27}$, of the cube of the number, minus $\frac{8}{27}$ of the cube, or $\frac{10}{27}$ of the cube of the number, equals 10. So the cube is 27 and the number is 3.

12. Two-thirds of the square of twice a number is equal to $\frac{4}{3}$ of the square of $\frac{1}{3}$ of the number, diminished by 3. What is the number?

Ans: $\frac{2}{3}$ of the square of twice a number = $\frac{8}{3}$ of the square of the number; $\frac{4}{3}$ of the square of $\frac{1}{3}$ of a number = 3 times the square of the number; 3 times the square minus $\frac{8}{3}$ of the square, or $\frac{1}{3}$ of the square = 3, the square = 9, the number = 3.

13. A boy spends $\frac{1}{2}$ of his money, + \$ $\frac{1}{2}$, then $\frac{1}{2}$ of the rest, + \$ $\frac{1}{2}$, and then had \$3. How much money did he have at first?

Ans: The first time he had $\frac{1}{2}$ of his money, - \$ $\frac{1}{2}$ left; the second time he had left $\frac{1}{2}$ of this, minus \$ $\frac{1}{2}$, which is $\frac{1}{4}$ of his money - \$ $\frac{3}{4}$, which equals \$3. So $\frac{1}{4}$ of his money = \$ $3\frac{3}{4}$, and his money = \$15.

14. C and D together have 20 sheep, and $\frac{1}{3}$ of C's number, + $\frac{1}{4}$ of D's, equals $\frac{1}{2}$ of C's: how many sheep does each own?

Solution—Since $\frac{1}{2}$ C's = $\frac{1}{3}$ C's + $\frac{1}{4}$ D's, $\frac{1}{4}$ D's = $\frac{1}{6}$ C's, or D's = $\frac{2}{3}$ C's; from which we find D has 8, and C, 12.

15. A boy spent $\frac{1}{2}$ of his money and \$2 more, and then spent \$2 more than $\frac{1}{2}$ of the rest, and then had \$2 left. How much did he have at first?

Ans: $\frac{1}{4}$ of his money, minus \$3 = \$2. So his money is \$20.

16. Sarah gave away $\frac{1}{3}$ of her sweaters, less $\frac{1}{3}$ of a sweater, and then gave away $\frac{1}{3}$ of the rest, lacking $\frac{1}{3}$ of a sweater, and then had $5\frac{4}{9}$ sweaters left. How many sweaters did she have at first?

Ans: We find $\frac{4}{9}$ of what she had at first = $5\frac{4}{9} - \frac{5}{9}$, or $\frac{44}{9}$ sweaters. So she had 11 at first.

17. A fish caught in the Conowingo weighs 8 pounds, and $\frac{2}{5}$ of the body, + $\frac{2}{3}$ of the head and tail, weigh as much as $\frac{4}{5}$ of the body. What is the weight of each part if the tail is $\frac{1}{3}$ as heavy as the head.

Ans: We find the weight of head and tail = $\frac{3}{5}$ weight of the body. So $\frac{8}{5}$ weight of the body = 8 lbs., the body weighs 5 lbs., and the head and tail 3 lbs.. Which, divided by the last condition, gives head $2\frac{1}{4}$ lbs., and tail $\frac{3}{4}$ lb.

18. A woman, when asked how many music students she had in her class, replied, that $\frac{2}{3}$ of the number multiplied by $\frac{3}{4}$ of the number is 9 more than the square of $\frac{1}{2}$ the number. How many students did she have?

Ans: $\frac{2}{3}$ of the number, multiplied by $\frac{3}{4}$ of the number, equals $\frac{1}{2}$ of the square of the number, and the square of $\frac{1}{2}$ of the number equals $\frac{1}{4}$ of the square of the number, and $\frac{1}{2}$ of the square, minus $\frac{1}{4}$ of the square, or $\frac{1}{4}$ of the square = 9. So the square is 36, and the number is 6.

19. A PERSON, when asked the hour of the day, replied that 2 hours ago the time past noon was $\frac{1}{3}$ of the time to midnight 2 hours so. What is the time.

Solution—We find $\frac{4}{3}$ of the time to midnight 2 hours so + 2 hours + 2 hours, equals 12 hours. So, $\frac{4}{3}$ of the time to midnight, 2 hours so, equals 8 hours, and $\frac{1}{3}$ of the time to midnight 2 hours

so, which was the time past noon 2 hours ago, equals $\frac{1}{4}$ of 8 hours, or 2 hours. So the time now is 2 hours + 2 hours, or 4 hours past noon, or 4 o'clock P.M.

20. A man went to a store and spent 20 cents, and then, losing $\frac{3}{4}$ as much as remained, had $\frac{1}{3}$ as much as he had at first, minus \$1. How much did he have at first?

Ans: By the conditions, $\frac{1}{4}$ of what he had at first, - 5 cents = $\frac{1}{3}$ of what he had at first, - 100 cents. So $\frac{1}{3}$ - $\frac{1}{4}$, or $\frac{1}{12}$ of what he had at first equals 100 - 5, or 95 cents. He had at first \$11.40.

21. A is 10 steps before B, and takes 2 steps while Ben takes 4, and 4 of A's steps equal 6 of B's. How many steps will each take before they are together?

Ans: We find that while Ben takes 6 steps a will take 3 steps, and since 6 of B's = 4 of A's, a will lose 1 step every time he takes 3 steps and Ben takes 6. So to lose 10 steps, the distance he was ahead, a will take 10 times 3, or 30 steps, and Ben will take 10 times 6, or 60 steps.

22. Said E to F, my age is 5 years more than yours, but 4 years ago my age was $\frac{1}{2}$ of what yours will be 4 years so. What was the age of each?

Ans: 1st.—E's age, minus 4 years, is $\frac{1}{2}$ of F's, + 2 years; but E's age = F's + 5 years, and E's age, minus 4 years = -F's age + 1 year. So F's age + 1 year = $\frac{1}{2}$ of F's, + 2 years. So $\frac{1}{2}$ of F's age is 1 year, F's age is 2 years, and E's 7 years.

2nd.—As by the first condition E was 5 years older than P, and by the second 8 years more are added to F's age than to E's, therefore, 4 years so, F will be 8 - 5, or 3 years older than E was 4 years ago; but since F's then was twice E's 4 years ago, the difference was equal to E's age 4 years ago, which was 3 years. So E is now 7 years, and F 2 years old.

23. A woman bought 10 meters of silk at the rate of \$4 a meter, but finding some of it damaged, for it she only paid \$1 a meter, and thus paid \$28. How many meters were damaged?

Ans: The loss was \$12 on the whole, and on each yard \$4 - \$3, or \$1. So there were 4 damaged yards.

24. Custer and D ran from the same point in the same direction, and when D had run 40 meters, $\frac{1}{9}$ of the distance Custer had run equaled the distance he was ahead of D. How much did C, in running 40 meters, gain on D?

Ans: C gains $\frac{1}{9}$ of the distance he runs. So in running 40 meters, he gains $\frac{1}{9}$ of 40 rods, or $4\frac{4}{9}$ meters

25. A boy bought some peaches at 4 cents each and 3 times as many pears at 2 cents each, and sold them all at 6 cents each, and thus gained 28 cents. How many of each did he buy?

Ans: We find he gained on 1 peach and 3 pears 14 cents. So, to gain 28 cents, he must sell 2 peaches and 6 pears.

26. My friend's watch loses 2 minutes in 3 hours, and mine gains 20 minutes a day; they were set with correct time yesterday noon, and are now half an hour apart. What time is it?

Ans: My friend's watch loses $\frac{2}{3}$ of a minute in an hour while mine gains $\frac{5}{6}$ of a minute in an hour. So in 1 hour after they were set they will be $\frac{2}{3} + \frac{5}{6}$, or $\frac{3}{2}$ of minute apart, and to be 30 minutes apart will require $30 \div \frac{3}{2}$, or 20 hours. So the time is 20 hours from yesterday noon, or 8 A.M. today.

27. A, B, and C can mow a field in 20 days, A and B in 30 days, and B and C in 40 days; after the three had worked 5 days, A and C finished it; in what time was it completed?

Ans: In 5 days they mowed $\frac{1}{4}$ of the field; C can do $\frac{1}{20} - \frac{1}{30}$, or $\frac{1}{60}$ in 1 day, and A can do $\frac{1}{20} - \frac{1}{40}$, or $\frac{1}{40}$ in 1 day. So A and C can do $\frac{1}{60} + \frac{1}{40}$, or $\frac{1}{24}$ in 1 day, and they can do $\frac{3}{4}$ in $\frac{3}{4} \div \frac{1}{24}$, or 18

days.

28. A person, when asked the time of day, replied that $\frac{2}{3}$ of the time past midnight, 2 hours ago, equaled $\frac{1}{2}$ of the time to midnight, 3 hours and 20 minutes so. What is the time.

Ans: By the conditions, $\frac{8}{5}$ of the time to midnight, 3 hours and 20 minutes so = $18\frac{2}{3}$ hours. So the time past midnight, 2 hours ago, is 7 hours; the time now, 9 A.M.

29. A pole whose length was 44 meters was broken into two unequal parts. If the shorter be increased by 3 meters, and the longer be diminished by 5 meters, the first will be $\frac{1}{2}$ as long as the second. What is the length of each part.

Ans: If the length of the shorter is increased by 3 feet, and the length of the longer be diminished by 5 feet, the length of the pole will be 42 feet; $\frac{3}{2}$ of the length of the longer = 42 feet. So the longer is $28 + 5$, or 33 feet; the shorter $14 - 3$, or 11 feet.

30. A staff whose length is 33 meters is in the air and water. And the length in the air, - 2 meters, equals 4 times the length in the water, + 6 meters. What is the length in the air.

Ans:

31. Two years ago Mr. Smith was 5 times as old as his son John will be 2 years so, and 3 years so his age will equal 15 times John's age 3 years ago. What is the age of each.

Ans: Mr. Smith's age minus 2 years = 5 times John's age, + 10 years, or Mr. Smith's age = 5 times John's age + 12 years. And by the second condition, Mr. Smith's age + 3 years = 15 times John's age, - 45 years, or Mr. Smith's age now = 15 times John's age, - 48 years. So 5 times John's age, + 12 years = 15 times John's age, - 43 years, or 10 times John's age = 60 years, John's age is 6 years, and Mr. Smith's age 42 years.

32. Arthur gave $\frac{1}{4}$ of his money, lacking 3 cents, to James, $\frac{1}{3}$ of the rest, lacking 2 cents, to Willie, and $\frac{1}{2}$ of the rest, lacking 1 cent, to Charles, and then had 8 cents left. What was Arthur's money before his gifts?

Ans: By the conditions of the problem, $\frac{1}{4}$ of his money + 3 cents = 8 cents. So $\frac{1}{4}$ of his money = 5 cents, and his money = 20 cents.

33. Jordan gave $\frac{1}{5}$ of his money, plus 4 cents, to John, $\frac{1}{4}$ of the rest, plus 3 cents, to George, and $\frac{1}{3}$ of what now remained, plus 2 cents, to Jackson, and found he had $\frac{1}{5}$ as much as at first. How much money did he have at first?

Ans: By the condition, $\frac{2}{5}$ of his money minus 6 cents = $\frac{1}{5}$ of his money. So he had 30 cents at first.

34. A woman, had two watches, bought a chain for \$20. If the chain is put on the silver watch, their value will be $\frac{1}{3}$ as much as the gold watch; but if it be put on the gold watch, they will be worth 7 times as much as the silver watch. What was the value of each watch?

Ans: By a condition of the problem, 7 times the value of the silver watch equals the value of the gold watch + \$20, but the gold watch equals 3 times the value of the silver watch, + \$60, therefore 7 times the value of the silver watch equals 3 times the value of the silver watch. + \$80. So 7 times - 3 times, or 4 times the value of the silver watch equals \$80; therefore once the value of the silver watch equals \$20, and the gold watch is worth

35. Two boats leave a wharf at the same time for the same point; it takes one boat 10 hours to reach it, and the other boat sails 5 times as fast going, and 10 times as fast returning. When do they meet?

Ans: The second boat would reach her destination in 2 hours, and the first boat would have 10

- 2, or 8 hours' journey left; in returning, the second boat would go over 10 times the distance the first goes, and together they would go 11 times the distance the first goes, or the distance passed by the first in 8 hours; the first goes $\frac{1}{11}$ of the distance, which requires $\frac{8}{11}$ of an hour. So they meet in $\frac{28}{11}$ hours from the time of starting.

36. Paid $\$34\frac{1}{2}$ for corn at $\$ \frac{3}{4}$, wheat at $\$1$, and oats at $\$ \frac{1}{2}$ a kg. Sold $\frac{2}{3}$ of the corn and $\frac{1}{2}$ of the wheat for 50 percent advance, gaining on the corn $\frac{2}{5}$ as much, as on the wheat, and on the sale the cost of the oats. How much of each did I buy?

Ans: By selling $\frac{2}{3}$ of the corn and $\frac{1}{2}$ of the wheat at 50 percent advance, I gained $\frac{1}{3}$ of the cost of the corn and $\frac{1}{4}$ of the cost of the wheat, and since I gained on the corn $\frac{2}{5}$ as much as on the wheat, $\frac{1}{3}$ of the cost of the corn is $\frac{2}{3}$ of $\frac{1}{4}$, or $\frac{1}{6}$ of the cost of the wheat. And the cost of the corn equals $\frac{1}{2}$ of the cost of the wheat; the whole gain was $\frac{1}{6} + \frac{1}{4}$, or $\frac{5}{12}$ of the cost of the wheat, which is the cost of the oats. So $\frac{6}{12} + \frac{5}{12} + 1\frac{2}{12}$, or $\frac{23}{12}$ of the cost of the wheat equals $\$34\frac{1}{2}$; the wheat cost $\$18$, the corn $\$9$, and the oats $\$7\frac{1}{2}$; therefore I bought 18 bushels of wheat, 12 of corn, and 15 of oats.

37. A person has two cups and a cover which weighs 30 ounces. If the first cup be covered it will weigh twice as much as the second, but if the second cup be covered it will weigh 3 times as much as the first. What is the weight of each cup?

Ans: The first cup weighs twice the second, - 30 oz., and the second cup weighs 3 times the first, - 30 oz., but 3 times the first equals 6 times the second, - 90 oz..

38. Company A and company B were hired by a construction company to excavate 100 meters of foundation for $\$10000$. And since the part which A was to dig was more difficult excavation than that which company B had to dig, it was agreed that A should receive 100 dollars per meter, and Bill 60 dollars per meter. Each company received $\$500$ for their work. How many meters did each dig?

39. A man, wishing to erect some buildings, concluded that if he built a store and a garage the store should cost $\frac{1}{3}$ of his money, but if he built a house and a garage, the garage should cost $\frac{1}{3}$ of his money. What was the cost of each, supposing he built all three, and their cost was $\$77,000$?

Ans: The garage costs twice as much as the store, and the house twice as much as the garage, or 4 times as much, as the store. So all three cost 7 times the price of the store, which is $\$77,000$, and the store cost $\$11,000$, the garage $\$22,000$, and the house $\$44,000$.

40. A, B, and C, considering the purchase of a machine, agreed that if A and B bought it, A should pay $\frac{2}{5}$ of the price, but if B and C bought it, B should pay $\frac{2}{5}$ of the price; then the three agreed to buy it together, when it was found that C paid $\$500$ more than A. What did the machine cost, and what did each pay?

Ans: By the first condition, A pays $\frac{2}{5}$ and B $\frac{3}{5}$, so $\frac{2}{3}$ of what B pays equals what A pays; by the second condition, B pays $\frac{2}{5}$ and C $\frac{3}{5}$, so $\frac{2}{3}$ of what C pays equals what B pays, and therefore A pays $\frac{2}{3}$ of $\frac{2}{3}$ of what C pays, or $\frac{4}{9}$ of what C pays; then $\frac{9}{9}$ of what C pays, minus $\frac{4}{9}$ of what C pays, or $\frac{5}{9}$ of what C pays, equals $\$500$. So C pays $\$900$, B $\$600$, A $\$400$.